

Lender Moral Hazard and Reputation in Originate-to-Distribute Markets

Andrew Winton* Vijay Yerramilli†

Preliminary and incomplete

Abstract

We analyze a dynamic model of originate-to-distribute lending in which a bank with significant liquidity needs makes loans and then sells them in the secondary loan market. There is no uncertainty about the bank's monitoring ability or honesty, but the bank may not have incentives to monitor the loan after it has been sold. We examine whether the bank's concern for its reputation, which is based on the number of recent defaults on loans it has originated, can maintain its incentives to monitor. In equilibrium, a bank that has had more recent defaults obtains a lower secondary market price on its current loan and monitors less intensively. Monitoring is more likely to be sustainable if the bank has greater liquidity needs or monitoring has a higher benefit-to-cost ratio; reputation is more valuable for greater liquidity needs, higher monitoring benefit-to-cost ratio, and higher base loan quality. If the bank can commit to retaining part of loans it makes, then a bank with worse reputation retains more of its loan. Competition from a rival lender makes it less likely that monitoring can be sustained and may cause a high-reputation bank to cede the loan to the rival. A temporary increase in loan demand (a "lending boom") makes it less likely that any monitoring can be sustained, especially for low-reputation banks.

*Carlson School of Management, University of Minnesota; email: awinton@csom.umn.edu

†C. T. Bauer College of Business, University of Houston; email: vyerramilli@bauer.uh.edu

Introduction

Traditional theories of financial intermediation emphasize that banks must hold the loans they make so as to maintain their incentives to screen and monitor them, but present-day lenders increasingly sell off the loans that they originate.¹ Although this “originate-to-distribute” (OTD) model can improve risk-sharing and the lender’s liquidity position, it also undermines the traditional mechanism for maintaining monitoring incentives. Up until the recent financial crisis, the typical response of market participants to such concerns was that the lender’s concern for its reputation would provide it with the incentives to monitor even after it had laid off its exposure to credit risk, but subsequent revelations of poor credit underwriting even by highly-reputed institutions cast doubt on this.² The natural question that follows is when and to what extent can such reputation concerns sustain monitoring by lenders?

In this paper, we address this question in a model of repeated OTD lending. A lender (“bank”) originates a loan; as the bank faces liquidity constraints, it wishes to sell the loan to investors without such constraints. Afterwards, the bank can monitor at a cost and reduce the loan’s chance of default. There is no uncertainty about the bank’s monitoring ability, but it cannot commit to monitor unless monitoring is incentive-compatible; i.e., there is no innately “honest” type of bank. It follows that, in a single-period setting, the bank would not monitor loans it sold off, reducing the expected value of its loans and overall welfare. But we consider a repeated setting, in which the bank faces a new borrower and a new set of investors each period. Now, investors can use the history of defaults on the bank’s loans as a noisy indication of the bank’s reputation for monitoring, i.e., to form their beliefs about the likelihood that the bank will monitor its current loan. We analyze the circumstances under which monitoring can be sustained by such reputation concerns, and the factors that may undermine monitoring.

In the spirit of Dellarocas (2005), we focus on equilibria where bank reputation depends on the number of its loans that defaulted over the most recent N periods.³ To fix ideas, we begin with the case where the bank is a monopolist and its reputation depends only on whether its most recent loan defaulted ($N = 1$); we then show that our analysis extends to using longer histories. The intuition for how such measures of reputation can sustain monitoring is as follows. Each period, the bank knows that if it does not monitor (“shirks”)

¹For traditional theories of delegated monitoring, see Leland and Pyle (1977), Diamond (1984) and Holmstrom and Tirole (1997). Gorton and Pennacchi (1995) are among the first to highlight the trend towards selling off originated loans.

²See Keys et al. (2010) and Purnanandam (2011) for empirical evidence that securitization led to lax screening in the mortgage market.

³Dellarocas (2005) explores reputation mechanism design in an online trading environment with pure moral hazard on the seller’s part and imperfect monitoring. He characterizes equilibria in which buyers condition their beliefs about the seller’s effort on the seller’s past performance history. We discuss differences between his model and ours below.

it saves the cost of monitoring, but this increases the likelihood that the loan will default, hurting the bank’s reputation next period. If the additional rents accruing to a higher future reputation are great enough, the bank will monitor more in the current period.

We show that if there is some monitoring in equilibrium, then the probability of monitoring is strictly higher if there have been fewer defaults in the last N periods. Thus, the secondary loan market price is also higher if the loan was originated by a bank that had fewer defaults recently. A “full monitoring” equilibrium in which the bank always monitors in the highest-reputation state but monitors with a lower probability in lower-reputation states is more likely to hold as the bank’s inter-temporal discount rate is lower, its liquidity needs are stronger, and as monitoring is less costly or has greater impact on default probability. An increase in these parameters also increases the value of a high reputation. In addition, the value of a high reputation also increases as the default probability in the absence of any monitoring (“baseline default probability”) decreases, because a decrease in the baseline probability of default lowers the probability of default by bad luck.

Several key features of our model are worth emphasizing here. First, as the bank has no innate type, the reputation mechanism does not reflect learning about the bank; instead, it operates purely through the threat of future punishment for poor performance.⁴ Second, while the use of past defaults is reminiscent of the “trigger strategy” equilibria (Green and Porter (1984), Abreu (1986)), our measure allows for multiple reputation states and more nuanced behavior: a low reputation now can improve later if subsequent defaults are fewer, and low-reputation banks may monitor with some intensity, albeit lower than that of high-reputation banks. Finally, because monitoring does not completely eliminate the possibility of default, defaults are a noisy signal of whether the bank has monitored or not. As a result, the second-best solution cannot support full monitoring indefinitely: defaults eventually occur, damaging bank reputation, which reduces the bank’s incentives to monitor. It is for this reason that reputation is less valuable when baseline default probability is high: this increases the likelihood that the bank’s reputation will be hurt even if it does monitor.

We then pursue a number of extensions of our base model. Suppose that the bank can commit to retain any fraction of the loan it makes in a given period. (Such a commitment might take the form of loan sale restrictions in the loan contract). Obviously, the bank could then commit to monitor simply by retaining a large fraction of the loan, but this would increase the bank’s liquidity costs. We show that as the bank’s recent loan performance is worse, it must retain a higher fraction of its current loan in order to guarantee monitoring. This is consistent with the empirical evidence in Gopalan et al. (2011) that a lead arranger that experiences large defaults is likely to retain a larger fraction of the loans that it underwrites in the subsequent year.

⁴As Dellarocas (2005) notes, because there is no learning about type in this sort of setting, a longer performance history will not improve incentives here.

Next, we examine the impact of competition. It is well known that in settings with pure moral hazard, a reputation mechanism can be sustained only if the agent obtains a reputation rent each period (Klein and Leffler (1981), Shapiro (1983)). Thus, by lowering the bank’s rent, competition may affect monitoring incentives. To capture this, we introduce the possibility of a rival lender appearing in any given period and competing with the incumbent bank for that period’s borrower. Such competition makes monitoring harder to sustain, as one would expect. Moreover, a high-reputation incumbent bank may shy away from situations where it must compete with a rival, whereas a low-reputation bank would not, leading to a form of Gresham’s Law. Intuitively, a low-reputation bank has nothing to lose from a default, and can potentially improve its reputation and future rents if the current loan does not default. By contrast, a high-reputation bank not only gets lower current period surplus (compared to monopoly) if it does compete with the rival and win the loan, but also risks damage to its reputation if the loan then defaults. If the current period surplus in the presence of a rival is sufficiently low, the high-reputation bank is better off ceding the loan to the rival and maintaining its reputation for the next period (when it may not face a rival).

Although our baseline model assumes that the bank faces constant loan demand each period, in reality, there are booms and busts in loan demand, and a reputed bank may be tempted to milk its reputation during a lending boom to earn a large but temporary surplus. Accordingly, we model how the bank’s monitoring incentives are affected by a temporary increase in the demand for loans (“lending boom”). If the bank chooses to increase its loan volume, its monitoring costs increase proportionally. This yields two key results. First, regardless of its current reputation, a bank that chooses to increase its loan volume during the boom will have no incentive to monitor during the boom: the marginal value of monitoring is linked to the future value of reputation, which in turn depends on normal loan volumes, but the savings from shirking on a larger-than-normal volume of loans more than offset this. Second, low-reputation banks are more likely than high-reputation banks to increase their lending volume and shirk on monitoring, because low-reputation banks have less to lose by relaxing lending standards.

Our paper is related to several recent papers. As noted above, technically, our paper is closest to Dellarocas’ (2005) model of reputation for internet sellers. In adapting this model to the interaction between bank reputation and monitoring in an OTD setting leads to a number of technical differences between his work and ours. First, we incorporate the role of seller liquidity needs, which in turn lets us examine tradeoffs between fractional loan retention and reputation as mechanisms that sustain monitoring. Second, we examine how competition affects a bank’s monitoring incentives and aggressiveness as a function of its reputation. We show that not only does competition lower reputation values and monitoring incentives, but it may also cause a high-reputation bank to cede the borrower to its rival. Third, given the experience of the recent financial crisis, we examine the effect

of temporary lending booms on the bank’s monitoring incentives.

A few recent financial intermediation papers use reputation models that, like ours, operate in a world of pure moral hazard. Bolton et al. (2007) examine whether a financial intermediary’s concern for its reputation can alleviate conflicts of interest between the intermediary and its customers (see also Bolton et al. (2009)). Both papers assume that the intermediary suffers an *exogenous reputation loss* when a lie told by the intermediary results in the customer purchasing an unsuitable financial product. By contrast, we endogenize the value of reputation and examine its sensitivity to a number of complicating factors. Bar-Isaac and Shapiro (2011) use a reputation model with pure moral hazard to understand how the value of reputation and the quality of ratings issued by credit rating agencies vary over the business cycle. One key difference from our paper is that they focus on grim-trigger-strategies where investors never purchase an investment rated by a rating agency that if found out to have issued a faulty good rating at *any* point in the past. As noted above, this does not allow the more nuanced behavior of our model, where reputations can be recovered. Moreover, their focus on ratings agencies abstracts from issues connected with loan origination, such as lender liquidity needs and loan retention decisions.

A much larger literature models reputation in settings where an agent’s actions are dictated by innate type as well as strategic concerns. Here, reputation arises from learning over time about an agent’s innate type, but the agent can adjust his or her behavior to affect the learning process (e.g., Kreps and Wilson (1982), Milgrom and Roberts (1982), Holmstrom (1999)). It is common to assume that the agent is either an “honest” type that is committed to acting in the first-best manner or a “strategic” type that always acts to maximize his or her utility. Diamond (1989) and Benabou and Laroque (1992) build on this literature to model reputation formation of borrowers in credit markets and financial gurus in the stock market, respectively. In such models, incentive problems are most severe for agents with short track records, and become less severe as the agent accumulates a good reputation following a good track record. By contrast, in our model of reputation with pure moral hazard, a long track record will not necessarily improve incentives because the bank has no innate type and can choose to either monitor or not monitor in each period. In fact, the value of reputation does not depend on the length of past performance history observed by borrowers and investors.

Among reputation papers using this “mixed” approach, the paper with the topic that is closest to ours is Hartman-Glaser (2011). Hartman-Glaser models a securitization game with reputation concerns, where the issuer can credibly signal the asset’s quality by retaining a portion of the asset. In his model, reputation concerns arise due to asymmetric information over the issuer’s innate preference for “honesty” (truthfully reporting a bad asset’s type). This difference affects his results. Although, like us, Hartman-Glaser finds that the issuer retains less of the asset when she has a higher reputation, the impact of rep-

utation on the issuer’s moral hazard problem is the opposite of ours: in his model, as the opportunistic issuer’s reputation improves, she *decreases* the probability that she truthfully reveals asset quality, whereas in our model, as the bank’s reputation improves, it *increases* the probability that it monitors.

Another related paper using this approach is Mathis et al. (2009). They examine a credit rating agency’s incentives to inflate ratings in a model of endogenous reputation, assuming the existence of an “honest” type that always reports truthfully. In addition, they assume that the ratings agency obtains some of its profits from another (unmodeled) line of business, and that this exogenous profit stream is lost if the ratings agency’s reputation is hurt. Like Hartman-Glaser (2011), they find that ratings agencies that only get income from ratings activities subject to moral hazard lie more as reputation increases. As the stream of profits from other non-strategic activities increases, the ratings agency’s incentives to behave honestly improves. By contrast, we show that even in a setting where all activities are subject to moral hazard, increased reputation can improve lender behavior.

The rest of the paper is organized as follows. We describe our baseline model in Section 1, and characterize the equilibrium in Section 2. In Section 3, we allow the bank to retain a portion of the loan on its books, and examine how the retention decision varies with the bank’s reputation. We introduce competition into the model in Section 4, and examine the impact of temporary lending booms in Section 5. Section 6 concludes the paper.

1 Baseline Model

Consider a monopolist long-run lender (“bank”) that exists for an infinite number of discrete periods, denoted $t = 0, 1, \dots$, and in each period, faces a new borrower and a new set of secondary loan market investors who only exist for one period. All agents are risk neutral. Let δ denote the bank’s per-period discount factor, which may reflect time value of money or the bank’s impatience; the higher the δ , the more patient the bank. The bank’s objective is to maximize the present value of its payoffs over the entire span of the game.

At the beginning of each period, a borrower obtains a loan of one unit from the bank to fund its project. By the end of the period, the project either succeeds, yielding X , or fails, yielding C , where $C < 1 < X$. The cash flows from the project are verifiable. Thus, default occurs only if the project fails; C represents the collateral value that can be seized in the event of a default. Let $R \leq X$ denote the loan repayment amount if the project succeeds. Thus, $R - C$ is the risky component of the loan that the bank obtains only if the project succeeds. As we describe below, R is determined in equilibrium.

The bank can improve loan outcomes by monitoring borrowers at a cost of $m > 0$. The project succeeds with probability p_L if the bank does not monitor, and with probability

$p_H = p_L + \Delta$ if it does, where $\Delta > 0$ denotes the impact of monitoring. Monitoring can be thought of as keeping an eye on the firm and enforcing covenants so as to keep the firm from engaging in moral hazard. The bank's monitoring effort is unobservable, and cannot be contracted upon. We refer to $1 - p_L$ as the "baseline default probability" because it denotes the probability of default in the absence of any monitoring.

The borrower will undertake the project only if its expected payoff from the project exceeds the value of its outside option, $u \geq 0$. Let q denote the borrower's conjecture regarding the probability with which the bank monitors. Therefore, the borrower's expected payoff from undertaking the project is $(p_L + q\Delta)(X - R)$. Because the bank is a monopolist, it will set the loan repayment at the lowest value at which the borrower is indifferent between undertaking the project and pursuing the outside option. Let $R(q)$ denote this indifference value; it must satisfy

$$(p_L + q\Delta)(X - R(q)) = u. \quad (1)$$

After the bank makes the loan but before it monitors, it experiences a liquidity shock that makes it value immediate cash at $1 + \beta$ per dollar for some $\beta > 0$. If instead it waits to collect loan payments, it only values those payments at 1 per dollar. We assume that there exists an active secondary loan market where the bank can sell the loan. Given the belief q regarding the bank's monitoring choice, the price of the loan in the secondary market is

$$\begin{aligned} P(q) &= (p_L + q\Delta)(R(q) - C) + C \\ &= (p_L + q\Delta)(X - C) + C - u, \end{aligned} \quad (2)$$

where the second equation follows from equation (1). For simplicity, we assume that the bank cannot credibly commit to hold a fraction of the loan because borrowers and investors cannot observe, or can observe only with significant delay, whether the bank has sold the loan or not. We relax this assumption in Section 3.

Assumption 1: $0 < \Delta < 1 - p_L$; monitoring lowers the probability of default but does not eliminate it completely.

Since default occurs with positive probability $1 - p_L - \Delta$ even if the bank monitors the loan, a default is not perfectly indicative of lack of monitoring on the bank's part. A decrease in the baseline probability of default $1 - p_L$ lowers the probability that the loan defaults by bad luck even when the bank monitors.

Observe that firm value net of monitoring cost is $(p_L + \Delta) \cdot (X - C) + C - m$ if the bank monitors, and $p_L(X - C) + C$ if it doesn't. Therefore, for monitoring to be socially optimal, it must be that $\Delta(X - C) > m$.

Assumption 2: $\Delta(X - C) > m$; monitoring is socially optimal.

Since monitoring cannot be contracted upon, it is clear that if the bank lived for only one period, it would not have any incentives to monitor the borrower once it has sold the loan; anticipating this, the investors will price the loan at $p_L (X - C) + C$. However, the same need not to be true for a long-lived bank if borrowers and investors could observe the performance of previous loans originated by the bank.

We assume that, for each of the past N periods, borrowers and investors observe whether the bank's loan in that period defaulted or not. Let d denote the number of defaults that the bank has caused in the previous N periods. We examine equilibria where borrowers and investors condition their beliefs about the bank's monitoring intensity based on the number of previous defaults d ; we denote the conjecture of market participants as $q(d)$. Hence, we refer to d as the bank's reputation. In such equilibria, the bank's current and past loan performance may affect its ability to originate and distribute loans in future periods. We examine whether and to what extent such reputation considerations can incentivize the bank to monitor the borrower.

Let

$$v \equiv (1 + \beta) \cdot [(p_L + \Delta) \cdot (X - C) + C - 1 - u] \quad (3)$$

denote the bank's total current period surplus if it monitors the borrower.

We characterized the secondary loan price in equation (2). If $P(q(d)) < 1$, then the bank will not originate the loan in the first place. It is convenient, but not necessary, to assume that $p_L (X - C) + C \geq 1 + u$, so that $P(q(d)) \geq 1$ for all d ; i.e., the bank never has to drop off completely from the loan market.

Assumption 3: $p_L (X - C) + C \geq 1 + u$; the borrower and the bank break even on the project even if the bank does not monitor.

2 Characterization of the Equilibrium

To simplify illustration, we initially set $N = 1$, i.e., we assume that borrowers and investors only observe whether the bank's most recent loan defaulted ($d = 1$) or not ($d = 0$). In Section 2.2, we examine the case where $N = 2$.

2.1 Equilibrium with $N = 1$

With $N = 1$, participants condition their beliefs about the bank's monitoring based on whether the bank's most recent loan defaulted ($d = 1$) or not ($d = 0$). We refer to a bank with $d = 0$ as the high-reputation bank, and the one with $d = 1$ as the low-reputation bank. Let $V(d)$ denote the expected discounted value of the bank's profits in equilibrium, as a

function of d .

Observe that, with $N = 1$, the bank's reputation at the end of the current period will depend only on whether the current loan defaults or not; its reputation would be $d = 1$ if the current loan defaults, and $d = 0$ otherwise. The monitoring decision affects the transition probabilities of the bank's reputation as follows. If the bank monitors the current borrower, then its reputation at the end of the current period is $d = 0$ with probability p_H , and $d = 1$ with probability $1 - p_H$. Therefore, ignoring the current period surplus from selling the loan (which is sunk when monitoring is chosen), the bank's expected payoff if it monitors is

$$V_{mon} = -m + \delta(p_H V(0) + (1 - p_H) V(1)). \quad (4)$$

On the other hand, if the bank shirks on monitoring, then its reputation at the end of the current period is $d = 0$ with probability p_L , and $d = 1$ with probability $1 - p_L$, which results in an expected payoff of

$$V_{shirk} = \delta(p_L V(0) + (1 - p_L) V(1)). \quad (5)$$

It is evident that the bank faces the following tradeoff in its choice of monitoring: Monitoring costs m , but it increases the probability of the bank being in the high reputation state by Δ , which is worth $\delta\Delta (V(0) - V(1))$ in present value terms. Therefore, for monitoring to be incentive compatible, it is necessary that $V_{mon} \geq V_{shirk}$, which is equivalent to

$$\Lambda \equiv V(0) - V(1) \geq \frac{m}{\delta\Delta}, \quad (6)$$

where Λ denotes the incremental value of the high reputation.

The current surplus from selling the loan is $S(d) = (1 + \beta) (P(q(d)) - 1)$, which can be written as

$$S(d) = q(d) \cdot A + B, \quad (7)$$

where

$$\begin{aligned} A &\equiv \Delta (1 + \beta) (X - C), \\ \text{and } B &\equiv (1 + \beta) (p_L (X - C) + C - 1 - u) \end{aligned} \quad (8)$$

Note that $S(d)$ is increasing in $q(d)$. Moreover, Assumption 3 ensures that $S(d) \geq 0$ even without any monitoring; i.e., $S(d) \geq 0$ for all d .

We can now write the Bellman equation:

$$\begin{aligned} V(d) = & S(d) - mq(d) + \delta q(d)(p_H V(0) + (1 - p_H) V(1)) \\ & + \delta(1 - q(d))(p_L V(0) + (1 - p_L) V(1)) \end{aligned} \quad (9)$$

Substituting for $S(d)$ from equation (7), and rearranging, yields:

$$V(d) = q(d) \cdot (A - m) + B + \delta((p_L + \Delta q(d)) \cdot \Lambda + V(1)) \quad (10)$$

Equation (10) states that the bank's expected value in equilibrium, $V(d)$, is the sum of two components: its current period surplus, $q(d) \cdot (A - m) + B$, and the present value of its expected value next period, $\delta[(p_L + \Delta q(d)) \cdot \Lambda + V(1)]$.

We have the following result.

Lemma 1 *In any monitoring equilibrium, $\Lambda = \frac{m}{\delta\Delta}$, i.e., the incentive compatibility condition (6) holds with equality. Moreover, $q(0) > q(1)$; the probability of monitoring is strictly higher if there was no default last period than if there was a default last period.*

Suppose $\Lambda > \frac{m}{\delta\Delta}$; then the bank will strictly prefer to monitor in both the high- and low-reputation states, such that $q(0) = q(1) = 1$. But if the bank monitors with the same intensity in both states, then it must be that $V(0) = V(1)$, which violates the incentive compatibility condition. Therefore, it must be that $\Lambda = \frac{m}{\delta\Delta}$. Making this substitution in the Bellman equation, it follows that $\Lambda = (q(0) - q(1))A$, which implies that $q(0) > q(1)$, because otherwise, monitoring is not incentive compatible. Combining with equation (1), an immediate implication of Lemma 1 is that $R(0) > R(1)$; the loan repayment is higher when the bank is in the high reputation state.

We now solve for a “full monitoring” equilibrium in which the bank always monitors the loan in the high-reputation state (i.e., $q(0) = 1$), but monitors with probability $q(1) = \hat{q} \in (0, 1)$ in the low-reputation state. Our next result characterizes the full monitoring equilibrium, and describes the conditions under which it is feasible. Define

$$V^* = \frac{1}{(1 - \delta)} \left(v - \frac{m(1 - p_L)}{\Delta} \right) \quad (11)$$

Proposition 1 *The full monitoring equilibrium described above is feasible if, and only if,*

$$m < \delta(1 + \beta)\Delta^2(X - C). \quad (12)$$

If Condition (12) is satisfied, then the equilibrium is characterized by

$$\hat{q} = 1 - \frac{m}{\delta(1+\beta)\Delta^2(X-C)}, \quad (13)$$

and the value function given by: $V(0) = V^*$ and $V(1) = V^* - \frac{m}{\delta\Delta}$.

Substituting $q(0) = 1$ and $q(1) = \hat{q}$ into the Bellman equation (10), we obtain that $V(0) - V(1) = (1 - \hat{q})A$. But, incentive compatibility requires that $V(0) - V(1) = \frac{m}{\delta\Delta}$. Therefore, it must be that $\hat{q} = 1 - \frac{m}{\delta\Delta A} = 1 - \frac{m}{\delta(1+\beta)\Delta^2(X-C)}$. For the equilibrium to be well-defined, it must be that $\hat{q} > 0$, which yields the feasibility condition (12) in the Proposition. It is easily verified that condition (12) is more likely to hold as monitoring cost m is lower, the discount factor δ is higher, the value of liquidity β is higher, the impact of monitoring Δ is higher, and project risk $X - C$ is higher.

Substituting $d = 0$ and $V(0) - V(1) = \frac{m}{\delta\Delta}$ in equation (10), and solving the equation for $V(0)$, yields $V(0) = V^*$; combining this with incentive compatibility yields the expression for $V(1)$. Note that, in a normal repeated game with perfect monitoring (i.e., if $p_L + \Delta = 1$), the value function (V^*) would be $(1 - \delta)^{-1}(v - m)$. In our setting, it is less because of the chance that, even if the bank monitors, there may be a default due to bad luck. It is easily verified that V^* increases as the impact of monitoring Δ increases, and decreases as the base default probability of the loan, $1 - p_L$, increases.

A key feature of our model is that default is a noisy signal of bank monitoring, because monitoring does not completely eliminate the possibility of default. Therefore, defaults eventually occur, damaging bank reputation. For a bank in the high reputation state, let n_{high} denote the number of periods it spends in the high reputation state before its reputation is damaged. Similarly, for a bank in the low reputation state, let n_{low} denote the number of periods it spends in the low reputation state before its reputation improves. Clearly, n_{high} and n_{low} are random variables whose probability distribution depends on the monitoring choices of the bank in the high and low reputation states, respectively. Our next result characterizes the expected durations in the high and low reputation states.

Lemma 2 *In a full monitoring equilibrium, if a bank is in the high reputation state, its expected duration in the high reputation state is $E[n_{high}] \equiv \frac{1}{1-p_L-\Delta}$ periods. On the other hand, if it is in the low reputation state, its expected duration in the low reputation state is $E[n_{low}] \equiv \frac{1}{p_L+\Delta\hat{q}}$ periods.*

Observe that $E[n_{high}]$ is increasing in p_L and Δ , and that $E[n_{high}] \rightarrow \infty$ as $p_L + \Delta \rightarrow 1$. On the other hand, after substituting for \hat{q} from equation (13), it is evident that $E[n_{low}]$ is increasing in monitoring cost m , and is decreasing in p_L , Δ , risky cash flow $(X - C)$, value of liquidity β , and the bank's discount rate δ .

2.2 Equilibrium with $N = 2$

We now consider the case where $N = 2$, i.e., we examine an equilibrium where borrowers and secondary market players condition their beliefs about the bank's monitoring or screening on the number of defaults d in the previous 2 periods. As we will see, matters are more complex now, so we introduce additional notation. Because the bank can experience two outcomes every period (default or no default), his past performance profile, x , can take on $2^2 = 4$ possible combinations. Denoting default and no default by 1 and 0, respectively, the four possible combinations are: 00, 01, 10 and 11, where the *left-most digit denotes the outcome in the most recent period*. Observe that in the binary system, these performance profiles correspond to $x = 0, 1, 2$ and 3 , respectively.⁵ The number of defaults, d , is obtained by summing the two digits in the performance profile; i.e.,

$$d(x) = \begin{cases} 2 & \text{if } x = 3 \\ 1 & \text{if } x = 1, 2 \\ 0 & \text{if } x = 0 \end{cases} \quad (14)$$

Observe that while both the performance profiles $x = 1$ and $x = 2$ have the same reputation today (because $d(1) = d(2) = 1$), the default is more recent in the $x = 2$ profile compared with the $x = 1$ profile. As we show below, this affects the transition in the bank's reputation over the next period.

The bank's next period performance profile and reputation will depend on whether or not its current period loan defaults. Given the current performance profile x , let $x^-(x)$ and $x^+(x)$ denote its performance profile next period following a default and no default, respectively. It is easily verified that

$$x^-(x) = \begin{cases} 2 & \text{if } x = 0, 1 \\ 3 & \text{if } x = 2, 3 \end{cases} \quad (15)$$

and

$$x^+(x) = \begin{cases} 0 & \text{if } x = 0, 1 \\ 1 & \text{if } x = 2, 3 \end{cases} \quad (16)$$

Let $V(x)$ denote the expected discounted value of the bank's profits in equilibrium, given the performance profile x . By the same intuition as in the $N = 1$ case, monitoring is incentive compatible for a bank with the performance profile x only if $V(x^+) - V(x^-) \geq \frac{m}{\delta\Delta}$. Using the performance profile transitions in equations (15) and (16), we obtain the following

⁵In Section A1 of Appendix A, we examine the possibility that market participants condition their beliefs about the bank's monitoring based on its performance history x instead of the number of defaults d .

incentive compatibility conditions:

$$V(0) - V(2) \geq \frac{m}{\delta\Delta}, \quad (17a)$$

$$V(1) - V(3) \geq \frac{m}{\delta\Delta} \quad (17b)$$

By the same logic as in Section 2.1, the Bellman equation can be written as

$$V(x) = q(d(x)) \cdot (A - m) + B + \delta(p_L + \Delta q(d(x))) \cdot (V(x^+) - V(x^-)) + \delta V(x^-) \quad (18)$$

Lemma 3 *In any monitoring equilibrium, the incentive compatibility conditions (17a) and (17b) bind with equality. Moreover, $q(0) > q(1) > q(2)$; the probability that a bank monitors in the current period is strictly decreasing in the number of defaults it has caused in the previous two periods.*

Although the proof of Lemma 3 is more involved than that of Lemma 1, the underlying intuition is very similar. For the incentive compatibility condition (17a) to hold, it is necessary that a bank with no past defaults monitor more intensively than a bank that has experienced one default in the past two period. Similarly, for condition (17b) to hold, it is necessary that a bank with only one past default monitor more intensively than one with two past defaults. These conditions can be met only if the two incentive compatibility conditions bind with equality.

As in Section 2.1, we now solve for a full monitoring equilibrium in which the bank fully monitors the loan in the highest reputation state ($q(0) = 1$), but monitors with lower probability in the lower reputation states such that the probability of monitoring is strictly decreasing in the number of past defaults. Specifically, let $q(d)$ be of the form $q(d) = 1 - d\theta$, where $\theta > 0$ is a constant that needs to be characterized. For such an equilibrium to exist, there must exist a $0 < \theta < \frac{1}{N}$ to ensure that the bank monitors with positive probability in all states.

Our next result describes the conditions under which the full monitoring equilibrium is feasible, and characterizes θ and the value function $V(x)$ for $x \in \{0, 1, 2, 3\}$.

Proposition 2 *The full monitoring equilibrium described above is feasible if, and only if,*

$$m < \frac{\delta}{2} (1 + \delta) \Delta^2 (1 + \beta) (X - C) \quad (19)$$

If condition (19) is satisfied, then the equilibrium is characterized by

$$\theta = \frac{m}{\delta (1 + \delta) \Delta^2 (1 + \beta) (X - C)} \quad (20)$$

and the value function given by: $V(0) = V^*$, $V(1) = V^* - \frac{m}{\delta(1+\delta)\Delta}$, $V(2) = V^* - \frac{m}{\delta\Delta}$, and $V(3) = V^* - \left(\frac{2+\delta}{1+\delta}\right) \frac{m}{\delta\Delta}$.

Using the Bellman Equation (18), and the fact that the incentive compatibility conditions bind with equality (Lemma 3), it is easy to show that $V(0) - V(2) = (1 + \delta)\theta A$. But incentive compatibility requires that $V(0) - V(2) = \frac{m}{\delta\Delta}$. Equating these two expressions and solving for θ yields the expression in equation (20). For the full monitoring equilibrium to be feasible, it must be that $\theta < \frac{1}{2}$, because otherwise $q(2) = 1 - 2\theta \leq 0$. Setting $\theta < \frac{1}{2}$ yields the feasibility condition in (19).

Taking R as given, condition (19) is more likely to be met when the monitoring cost m is low, when the impact of monitoring Δ is high, when the value of liquidity β is high, and when the bank's discount factor δ is high. Also note that, because $\frac{1+\delta}{2} < 1$, condition (19) is more stringent than the equivalent condition for the $N = 1$ case. This is because it is easier for the bank to regain the highest reputation state following a default when its reputation depends only on the most recent loan performance (i.e., when $N = 1$); all it requires is that the current loan not default. On the other hand, regaining the highest reputation state following a default is more difficult if reputation depends on the performance in the previous two periods (i.e., $N = 2$); now the bank has to survive two periods without experiencing another default. Therefore, the bank's incentive to monitor are stronger in the $N = 1$ case compared to the $N = 2$ case, which explains why the full monitoring equilibrium is more likely to be feasible with $N = 1$. (In general, all else equal, feasibility of the reputation equilibrium is less likely as N increases.)

Substituting $d = 0$ and $V(0) - V(2) = \frac{m}{\delta\Delta}$ in equation (18), and solving the equation for $V(0)$ yields $V(0) = V^*$, where V^* is as defined in equation (11). The expressions for $V(1)$, $V(2)$ and $V(3)$ are obtained using the incentive compatibility conditions and the Bellman equation.

3 Loan Retention and Reputation

In the base model, we assumed that the bank could not credibly commit to retain a fraction of the loan on its balance sheet. In this section, we depart from the base model and assume that the bank can credibly commit to retain a fraction $\alpha \in [0, 1]$ of the loan on its books. An immediate implication of this assumption is that monitoring may be sustained even in a one-period setting without any reputational considerations if α is sufficiently high, specifically if $\alpha\Delta(R - C) \geq m$. Let

$$\alpha_{sp} \equiv \frac{m}{\Delta \left(X - C - \frac{u}{p_L + \Delta} \right)} \quad (21)$$

denote the critical threshold level of α above which monitoring can be sustained in a single period setting. We now explore how reputation, α and monitoring interact in a multi-period setting.

In a multi-period setting, the market's beliefs about bank monitoring will depend both on the bank's reputation d and α . Let $q(d, \alpha)$ denote this belief. Given reputation d , let $\alpha(d)$ denote the fraction of the loan that the bank will hold in equilibrium, and let $V(d)$ denote the expected discounted value of the bank's profits in equilibrium. As before, denote $\Lambda = V(0) - V(1)$. If a bank with reputation d holds a fraction α of the loan with repayment value R , then its payoffs from monitoring and shirking, respectively, are

$$V_{mon}(d) = -m + \delta [p_H V(0) + (1 - p_H) V(1)] + \alpha \cdot (p_H R + (1 - p_H) C) \quad (22)$$

and

$$V_{shirk}(d) = \delta [p_L V(0) + (1 - p_L) V(1)] + \alpha \cdot (p_L R + (1 - p_L) C), \quad (23)$$

Therefore, for there to be some monitoring in equilibrium, it must be that

$$\delta \Delta \Lambda + \alpha \Delta (R - C) \geq m \quad (24)$$

We know one more thing about α and the value function $V(\cdot)$. In a reputation equilibrium, $\alpha < \alpha_{sp}$. Hence, it must be that $\alpha \Delta (R - C) < m$, which in turn, implies that $\delta \Delta \Lambda > 0$.

In equilibrium, the market conjectures the bank's monitoring perfectly; i.e., $q(d, \alpha) = q$. Therefore, by the logic established in equation (1), the loan repayment value must satisfy $R = R(q) = X - \frac{u}{p_L + \Delta q}$ (where we have suppressed the arguments of q for convenience). Substituting for $R(q)$ in condition (24) yields the following condition which must be satisfied in equilibrium:

$$\delta \Delta \Lambda + \alpha \Delta \left(X - C - \frac{u}{p_L + \Delta q} \right) \geq m \quad (25)$$

Next, let us examine the bank's choice of α . Because $P(q) = (p_L + \Delta q)(R(q) - C) + C$, the bank's current period surplus $S(\alpha, d) = (1 + \beta) \cdot ((1 - \alpha)P(q) - 1) + \alpha P(q)$. Note that $P(q)$ is set after the bank announces α . Hence, we can substitute $P(q) = (p_L + \Delta q)(X - C) + C - u$, which allows us to rewrite $S(\alpha, d)$ as follows:

$$S(\alpha, d) = (1 + \beta(1 - \alpha)) \cdot [(p_L + \Delta q)(X - C) + C - u] - (1 + \beta). \quad (26)$$

The value function V can then be written as follows:

$$V(\alpha, d) = S(\alpha, d) - mq + \delta (p_L + \Delta q) \cdot \Lambda + \delta V(1) \quad (27)$$

As $S(\alpha, d)$ is decreasing in α , the bank will choose the lowest α at which monitoring is incentive compatible.

Lemma 4 *In any monitoring equilibrium, the incentive compatibility constraint (24) binds with equality for all d . Moreover, $q(0) > q(1)$ and $\alpha(0) < \alpha(1)$; the bank monitors more intensively and holds a smaller fraction of the loan in the high-reputation state.*

As the bank values immediate liquidity at $\beta > 1$, it incurs a liquidity cost by retaining a fraction $\alpha > 0$ of the loan. Therefore, in equilibrium, it will hold the lowest possible α at which the incentive compatibility constraint (24) binds with equality, because otherwise it can improve its expected value by retaining a slightly lower fraction $\hat{\alpha} = \alpha - \varepsilon$ while still maintaining the incentives to monitor.

Next, if the condition (25) holds with equality, then it must be that

$$\alpha(d) = \frac{m - \delta\Delta\Lambda}{\Delta \left(X - C - \frac{u}{p_L + \Delta q(d)} \right)}, \quad (28)$$

i.e., the higher the q , the lower is α .

Moreover, the Bellman equation can be rewritten as follows (see the proof of Lemma 4 for details):

$$\begin{aligned} V(d) &= (1 - \alpha(d)) \cdot \{(1 + \beta) [(p_L + \Delta q(d))(X - C) - u] + \beta C\} \\ &\quad + C - (1 + \beta) + \frac{mp_L}{\Delta} + \delta V(1) \end{aligned} \quad (29)$$

It is evident from equation (29) that $V(\cdot)$ is increasing in q and decreasing in α . Therefore, for the incentive compatibility condition to be satisfied, it is necessary that $q(0) > q(1)$, which implies that $\alpha(0) < \alpha(1)$.

4 Reputation with Competition

In our baseline model, we assumed that the bank is a monopolist. As the bank captures the entire surplus from the loan, this assumption effectively guarantees the bank a stream of positive rents if it makes and sells loans. In this section, we allow for the possibility of competition from other lenders, and examine the impact on the bank's incentives to maintain a reputation for monitoring.

To simplify matters, we look at the one-period reputation model of Section 2.1. The incumbent bank begins with a reputation $d \in \{0, 1\}$. In each period, there is a probability λ that another bank (the rival) will compete for that period's borrower. To rule out collusion,

we assume that if a rival enters in a subsequent period, it is a different bank. Also, rivals are assumed to have no reputation and to sell off their loans, so they have no incentive to monitor. We relax this restriction in Section B1 of Appendix B, where we allow competition from a long-lived rival with reputation $d \in \{0, 1\}$. Finally, in the event the incumbent and the rival offer the borrower the same rate, it will choose to go with the incumbent.

It follows that, if a rival appears, it will bid the loan face value R down to the point at which it breaks even; i.e., $p_L R + (1 - p_L)C = 1$. Let $R_{rival} \equiv \frac{1-C}{p_L} + C$ be this break-even rate. On the other hand, if a rival does not appear, the incumbent bank can set $R = R(q)$ as specified in equation (1).

Suppose the bank's record from last period was d . Let $V(d, rival)$ denote the bank's expected discounted profits if a rival is currently present, and $V(d, none)$ denote its expected discounted profits if no rival is present. Given that the rival arrives with probability λ , the expected value of having a reputation of d given that a rival may or may not appear this period is

$$V(d) \equiv \lambda V(d, rival) + (1 - \lambda)V(d, none) \quad (30)$$

As in the baseline setting, the main impact of monitoring is to increase the odds of having a good record in the future at a cost of m , and this impact is independent of the bank's current reputation. For monitoring to be incentive compatible, it is necessary that $\Lambda \equiv V(0) - V(1) \geq \frac{m}{\delta\Delta}$.

We know one more thing about the bank's value function. A bank that does not face a rival can do no worse than if a rival were present, because it can always imitate what it would do if a rival were present. Therefore, we have $V(d, none) \geq V(d, rival)$.

We now turn to the Bellman equation for the bank's value function. Let $q(d)$ be the probability that a bank with reputation d monitors in equilibrium. If the bank faces a rival, it can compete for the loan, get it, and lock in current surplus less expected monitoring costs, plus expected discounted future profits. Since the lending rate with a rival is R_{rival} , current surplus $S(R, q(d))$ will be equal to

$$\begin{aligned} S(R_{rival}, q(d)) &= (1 + \beta) \{ [p_L + q(d)\Delta] (R_{rival} - C) + C - 1 \} \\ &= (1 + \beta) q(d) \Delta \cdot \frac{1 - C}{p_L}. \end{aligned} \quad (31)$$

Therefore, the the expected discounted profits of the bank if it competes with the rival bank and gets the loan is

$$\begin{aligned} V_{compete}(d) &= S(R_{rival}, q(d)) - mq(d) + \delta q(d) \cdot (p_H \Lambda + V(1)) \\ &\quad + \delta (1 - q(d)) \cdot (p_L \Lambda + V(1)) \end{aligned} \quad (32)$$

Substituting for $S(R_{rival}, q(d))$ from equation (31), the above expression simplifies to

$$V_{compete}(d) = q(d) \left((1 + \beta) \Delta \cdot \frac{1 - C}{p_L} - m + \delta \Delta \Lambda \right) + \delta (p_L \Lambda + V(1)) \quad (33)$$

Alternatively, the incumbent bank can choose to wait until the following period, preserving its current reputation for the next period but earning no current surplus, thus earning a value $\delta V(d)$. It follows that the Bellman equation when the bank faces a rival is given by

$$V(d, rival) = \max \{ \delta V(d), V_{compete}(d) \} \quad (34)$$

We have the following result:

Lemma 5 *In any monitoring equilibrium, when faced with a rival,*

1. *A bank in the low reputation state ($d = 1$) always competes for the current period loan.*
2. *A bank in the high reputation state ($d = 0$) may not compete for the current period loan; a sufficient condition is*

$$(1 + \beta) \Delta \cdot \frac{1 - C}{p_L} \leq \frac{m}{\Delta} (1 - p_L). \quad (35)$$

A bank whose reputation has been damaged by a default in the previous period ($d = 1$) has nowhere to go but up: if it makes the loan, it has a chance of improving its reputation, and thus, its expected future profits. While the current surplus when the rival is present might not offset the costs of monitoring, the expected gain in future profits more than offsets this cost. By contrast, a high-reputation bank ($d = 0$) that competes for the loan has a chance of hurting its reputation: even if it monitors with probability 1, there is a chance the loan may default. If the current surplus when the rival is present is sufficiently low, getting the loan does not offset the costs of monitoring, and possible loss of reputation. In this case, the bank chooses to wait until next period, preserving its reputation for the chance that it can lend when no rival is present.

The upshot is that the presence of a rival not only decreases rents (since $V(d, rival) \leq V(d, none)$), but may drive more reputable banks out of the market. This is more likely when the impact of monitoring Δ is low, or the cost of monitoring m is high. Also, an increase in collateral value C reduces current surplus from monitoring in the presence of the rival, making it more likely that the reputable bank is driven out. However, an increase in the loan's base chance of default $1 - p_L$ has two offsetting effects: on the one hand, it increases current surplus, but on the other hand this drives up the chance of possibly losing reputation, and thus, having reduced future profits.

By contrast, the bank always lends when it does not face a rival. If it chooses not to lend, its expected profits equal $\delta V(d)$. If this were optimal, we would have $V(d, none) = \delta V(d)$. But $V(d, none) > V(d)$ unless current surplus with no rival is zero, which is only true if the bank does not monitor at all. Thus, as long as there is some monitoring in equilibrium, the bank lends when it does not face a rival. Indeed, even if the bank does not monitor, it is possible it will earn a positive surplus and thus choose to lend; this occurs when $p_L(X - C) + C > 1$.

Lemma 6 *In any monitoring equilibrium, $\Lambda = \frac{m}{\delta\Delta}$, i.e., the incentive compatibility condition holds with equality. Moreover, $q(0) > q(1)$; the probability of monitoring is strictly higher if there was no default last period than if there was a default last period.*

The intuition behind Lemma 6 is very similar to that behind Lemma 1 in the baseline model. If $\Lambda > \frac{m}{\delta\Delta}$, then $q(0) = q(1) = 1$ because the bank will strictly prefer to monitor regardless of d . However, then, it can be shown that the difference in expected discounted profits between the high and low reputation states will not be high enough for monitoring to be incentive compatible. Therefore, in any monitoring equilibrium, it must be that $\Lambda = \frac{m}{\delta\Delta}$ and that $q(0) > q(1)$.

As in the previous section, we now solve for the full monitoring equilibrium in which the high-reputation bank always monitors the loan (i.e., $q(0) = 1$) while the low-reputation bank monitors with probability $q(1) = \hat{q} \in (0, 1)$. Note that if $q(0) = 1$, then using equation (33) and the fact that $\Lambda = \frac{m}{\delta\Delta}$, we obtain that

$$V_{compete}(0) - \delta V(0) = (1 + \beta) \Delta \cdot \frac{1 - C}{p_L} - \delta \Lambda (1 - p_L). \quad (36)$$

Therefore, the condition (35) in Lemma 5 becomes both necessary and sufficient for the incumbent bank with high reputation to exit in the face of a rival.

Our next result characterizes the full monitoring equilibrium and describes the conditions under which it is feasible. Define

$$V_{no.exit}^* = \frac{1}{(1 - \delta)} \left[\lambda \frac{(1 + \beta) \Delta (1 - C)}{p_L} + (1 - \lambda) v - \frac{m(1 - p_L)}{\Delta} \right] \quad (37)$$

Proposition 3 (Equilibrium with competition):

1. *Suppose Condition (35) does not hold. Then the full monitoring equilibrium exists if, and only if,*

$$m < \delta \Delta^2 (1 + \beta) \left[\lambda \cdot \frac{1 - C}{p_L} + (1 - \lambda) (X - C) \right] \quad (38)$$

If this condition is met, a high-reputation bank always competes for the loan and fully monitors the loan, while a low-reputation bank monitors with probability

$$\hat{q}_{no_exit} = 1 - \frac{m}{\delta\Delta^2(1+\beta)} \cdot \frac{1}{\left[\lambda \cdot \frac{1-C}{p_L} + (1-\lambda)(X-C)\right]} \quad (39)$$

The expected value of having a high reputation is $V(0) = V_{no_exit}^*$, while the expected value of having a low reputation is $V(1) = V_{no_exit}^* - \frac{m}{\delta\Delta}$.

2. Suppose Condition (35) holds. Then the full monitoring equilibrium exists if, and only if,

$$m < \frac{\delta\Delta^2(1+\beta)(1-\lambda)(X-C)}{(1-\delta\lambda(1-p_L))} \quad (40)$$

If this condition is met, a high-reputation bank always monitors the loan but exits when faced with a rival, while a low-reputation bank monitors with probability

$$\hat{q}_{exit} = \frac{(1-\lambda)(X-C) - \frac{m}{\delta\Delta^2(1+\beta)}(1-\delta\lambda(1-p_L))}{\left[\lambda \cdot \frac{1-C}{p_L} + (1-\lambda)(X-C)\right]} \quad (41)$$

The expected value of having the high reputation is $V(0) = (1-\lambda)V^*$, while the expected value of having the low reputation is $V(1) = (1-\lambda)V^* - \frac{m}{\delta\Delta}$.

We solve for the equilibrium along the same lines as in the baseline model. First, we use the Bellman equation to obtain an expression for $\Lambda = V(0) - V(1)$ in terms of \hat{q} ; this expression will depend on whether or not condition (35) is met, i.e., whether or not a high-reputation bank exits in the face of a rival. Then, we use the incentive compatibility condition, $\Lambda = \frac{m}{\delta\Delta}$ to solve for \hat{q} in terms of the model parameters. The feasibility condition (either (38) or (40)) is obtained by noting that \hat{q} must be positive for the equilibrium to be well defined.

As in the baseline model, it is easily verified that both the feasibility conditions are more likely to hold as monitoring cost m is lower, the discount factor δ is higher, the value of liquidity β is higher, the impact of monitoring Δ is higher, and credit exposure $X - C$ is higher. Moreover, the feasibility conditions are less likely to hold as the chance of competition λ increases, and the base probability of success p_L increases, because an increase in λ or p_L decreases the current period surplus by lowering the loan's repayment value.

Finally, the possibility of competition also lowers the expected discounted value of the bank's profits, in both the high and low reputation states. As can be seen, the expected value of having the high reputation ($d = 0$) decreases from V^* to either $V_{no_exit}^*$ or $(1-\lambda)V^*$, depending on the bank's response to competition in the high reputation state.

To summarize, competition has two effects. First, as first noted by Klein and Leffler (1981), it erodes the incumbent bank’s rents, reducing incentives for monitoring and overall value. Second, it may tempt incumbents with good reputations to cede borrowers to rivals in the hopes of finding future borrowers that do not have access to rivals. Intuitively, if the incumbent competes for the loan, it will receive low rents this period and, even if it monitors, a chance of having a default, worsening its record. If instead it waits, it avoids any deterioration in its reputation and preserves the option of going after a borrower in the future if it does not face competition then.

5 Monitoring Incentives during Lending Booms

Until now, we have assumed that the bank faces a constant demand for loans each period, which we normalize to 1. In this section, we investigate how the bank’s monitoring incentives are affected by a one-time short-lived increase in the demand for loans. For the analysis in this section, we revert to our baseline setting with a monopolistic bank.

Suppose that there are periods in which the bank’s loan demand increases to $\gamma > 1$ (a “lending boom”) before reverting to the normal level of 1 in the next period. When it faces a higher loan demand, the bank can either increase its lending to γ or continue to lend only 1 as always. We do not allow for any randomization.⁶ If it increases its lending volume, then the success returns and collateral are γX and γC , respectively, and the total monitoring cost is γm . Purely for convenience, we assume that the bank’s lending volume is not observed by the market participants. The implication of this assumption is that the bank’s reputation in the period following the lending boom does not depend on its volume of lending during the boom period. We relax this assumption in Section B2 of Appendix B, where we show that the qualitative results from this section continue to hold even if the bank’s lending volume is observable, provided γ is large enough.

As we have already characterized the equilibrium when the bank loans one unit per period (see Section 2.1), we now focus attention on the case where the bank lends γ in the current period. Given reputation d , let $V_\gamma(d)$ denote the expected discounted value of the bank’s profits if it increases its quantum of lending to γ in the current period, and reverts back to lending one unit per period from the next period onward.

Excluding current surplus from lending γ , which is sunk when monitoring is chosen, the bank’s continuation payoff from monitoring is

$$V_{mon,\gamma} = -\gamma m + \delta [p_H V(0) + (1 - p_H) V(1)], \quad (42)$$

⁶For instance, it may be that during such boom periods, the borrower has a choice between implementing two indivisible projects – one that requires an investment of 1 unit, and a larger project that requires an investment of γ units. The bank can then choose to finance either of these two projects.

while its continuation payoff from shirking is

$$V_{shirk,\gamma} = \delta [p_L V(0) + (1 - p_L) V(1)] \quad (43)$$

Note that, in writing the expressions for $V_{mon,\gamma}$ and $V_{shirk,\gamma}$, we have exploited the assumption that the bank's lending volume during the boom period is not observed by market participants. Therefore, its reputation in the next period only depends on whether its current loan defaults or not. In Section B1 in Appendix B, we analyze the more general case where the bank's reputation may also depend on its lending volume during the boom period.

We showed in Section 2 that monitoring can be sustained in equilibrium only if $\delta\Delta(V(0) - V(1)) = m$. But then,

$$V_{mon,\gamma} - V_{shirk,\gamma} = \delta\Delta(V(0) - V(1)) - \gamma m < 0,$$

because $\gamma > 1$. In other words, a bank that experiences a one-time increase in lending volume to γ will not have enough incentives to monitor the loan. Investors will anticipate this and price these loans at $\gamma(p_L(X - C) + C - u)$. Hence, the bank's current period surplus from lending γ is

$$S_\gamma = (1 + \beta)\gamma \cdot [p_L(X - C) + C - 1 - u]. \quad (44)$$

Note that the current surplus does not depend on d because $q_\gamma(d) = 0$ for $d = 0, 1$. Therefore, it must be that $V_\gamma(0) = V_\gamma(1) = V_\gamma$, where

$$\begin{aligned} V_\gamma &= S_\gamma + \delta [p_L V(0) + (1 - p_L) V(1)] \\ &= S_\gamma + \delta \left[V^* - (1 - p_L) \frac{m}{\delta\Delta} \right], \end{aligned} \quad (45)$$

where the last equation follows by substituting $V(0) = V^*$ and $V(1) = V^* - \frac{m}{\delta\Delta}$.

We have the following result:

Proposition 4 *Suppose the feasibility condition (12) for the full monitoring equilibrium characterized in Proposition 1 is met, and suppose there is a one-time increase in loan demand to $\gamma > 1$. Then, there exist thresholds γ_l and γ_h with $\gamma_h > \gamma_l > 1$ such that,*

1. *If $\gamma \geq \gamma_h$, the bank increases its lending to γ and shirks on monitoring in both the high and low reputation states.*
2. *If $\gamma_h > \gamma \geq \gamma_l$, the bank increases its lending to γ and shirks on monitoring only in the low reputation state, while it lends one unit and fully monitors the loan in the high*

reputation state.

- If $\gamma < \gamma_l$, the bank lends only one unit, fully monitors in the high reputation state, and monitors with probability \hat{q} in the low reputation state.*

The threshold γ_l decreases as the cost of monitoring (m) increases, and increases as impact of monitoring (Δ), loan exposure ($R - C$), discount factor (δ) and the value of liquidity (β) increase.

While deciding whether to loosen its credit standards and lend γ , a bank trades off the higher surplus from lending γ with the cost of a loss of its reputation when the loan defaults. A bank with reputation d will choose to loosen its credit standards and lend γ if, and only if, $V_\gamma \geq V(d)$. (Here, we assume that the bank will choose to lend γ if it is indifferent between the two options.) Substituting for V_γ and $V(d)$, and simplifying, yields the result in Proposition 4. The thresholds γ_h and γ_l are characterized in the proof of Proposition 4.

Not surprisingly, a bank in the low reputation state is more likely to loosen its credit standards in order to obtain a higher surplus in the current period ($\gamma_l < \gamma_h$). This is because, given that it is in the low reputation state ($d = 1$), it has less to lose from another default in the current period. The comparative statics on γ_l indicate that loosening of credit standards is less likely (i.e., γ_l is higher) when the cost of monitoring m is low, the impact of monitoring Δ is high, the bank's credit risk exposure $X - C$ is high, its discount rate δ is high, and the value of liquidity β is high.

6 Concluding Remarks

In this paper, we analyze a dynamic model of OTD lending in which there is no uncertainty about the monitoring ability or honesty of the bank that originates the loan, but the bank may not have incentives to monitor after it has sold the loan. In this setting, we examine whether the bank can maintain its incentives to monitor out of concern for its reputation, which is endogenously determined. In the spirit of Dellarocas (2005), we examine equilibria where the market participants' beliefs regarding the bank's monitoring choice (i.e., the bank's reputation) depends on the number of defaults in the bank's recent performance history. As the bank's past performance does not contain any information regarding the bank's monitoring choice, the reputation mechanism works by punishing the bank for defaults. In equilibrium, a bank that has caused more defaults in the past obtains a lower secondary market price on its current loan, monitors less intensively, and retains a larger fraction of the loan on its books.

We then examine how the interplay between reputation and monitoring is affected by the arrival of a rival lender that competes with the incumbent bank for that period's loan. Not

surprisingly, competition lowers rents across the board, and therefore, lowers the likelihood that monitoring can be sustained in equilibrium. However, a more interesting result is that, when faced with a rival, the high reputation bank may cede the loan to the rival in a bid to maintain its reputation for the next period when it may not face a rival. Thus, competition may actually end up driving high-reputation banks from the market.

Finally, we examine how the bank's monitoring incentives are affected by a one-time short-lived increase in the demand for loans. We show that monitoring is less likely to be sustained for loans originated during such lending booms. This is because monitoring incentives are set by the incremental value of ending the current period with a high reputation, which depends on the regular level of future loan demand, that is lower than the demand during the boom period. We also show that low reputation banks, that have less to lose from fresh defaults, are more likely to increase their lending volume while shirking on monitoring.

Overall, our analysis sheds light on the effectiveness of reputation mechanisms in sustaining monitoring in OTD markets, and how monitoring incentives are affected by competition among lenders and by the occurrence of lending booms.

References

- Abreu, D. (1986). Extremal equilibria of oligopolistic supergames. *Journal of Economic Theory* 39, 191–225.
- Bar-Isaac, H. and J. Shapiro (2011). Ratings quality over the business cycle. Working Paper, New York University.
- Benabou, R. and G. Laroque (1992). Using privileged information to manipulate markets: Insiders, gurus, and credibility. *Quarterly Journal of Economics* 107(3), 921–958.
- Bolton, P., X. Freixas, and J. Shapiro (2007). Conflicts of interest, information provision, and competition in the financial services industry. *Journal of Financial Economics* 85, 297–330.
- Bolton, P., X. Freixas, and J. Shapiro (2009). The credit ratings game. NBER Working Paper.
- Dellarocas, C. (2005). Reputation mechanism design in online trading: Environments with pure moral hazard. *Information Systems Research* 16(2), 209–230.
- Diamond, D. (1984). Financial intermediation and delegated monitoring. *Review of Economic Studies* 51, 394–414.
- Diamond, D. (1989). Reputation acquisition in debt markets. *Journal of Political Economy* 97(4), 826–862.
- Gopalan, R., V. Nanda, and V. Yerramilli (2011). Does poor performance damage the reputation of financial intermediaries? evidence from the loan syndication market. Forthcoming, *Journal of Finance*.
- Gorton, G. and G. Pennacchi (1995). Banks and loan sales: Marketing non-marketable assets. *Journal of Monetary Economics* 35, 389–411.
- Green, E. J. and R. H. Porter (1984). Noncooperative collusion under imperfect price information. *Econometrica* 52(1), 87–100.
- Hartman-Glaser, B. (2011). Reputation, signaling, and security issuance. Working Paper, Duke University.
- Holmstrom, B. (1999). Managerial incentive problems: A dynamic perspective. *Review of Economic Studies* 66, 169–182.
- Holmstrom, B. and J. Tirole (1997). Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics* CXII, 663–691.

- Keys, B. J., T. Mukherjee, A. Seru, and V. Vig (2010). Did securitization lead to lax screening? evidence from subprime loans. *Quarterly Journal of Economics* 125, 307–362.
- Klein, B. and K. B. Leffler (1981). The role of market forces in assuring contractual performance. *Journal of Political Economy* 89, 615–641.
- Kreps, D. M. and R. Wilson (1982). Reputation and imperfect information. *Journal of Economic Theory* 27, 253–279.
- Leland, H. E. and D. H. Pyle (1977, May). Informational assymetries, financial structure and financial intermediation. *Journal of Finance* 32.
- Mathis, J., J. McAndrews, and J.-C. Rochet (2009). Rating the raters: Are reputation concerns powerful enough to discipline rating agencies? *Journal of Monetary Economics* 56, 657–674.
- Milgrom, P. and J. Roberts (1982). Predation, reputation, and entry deterrence. *Journal of Economic Theory* 27, 280–312.
- Purnanandam, A. (2011). Originate-to-distribute model and the subprime mortgage crisis. *Review of Financial Studies* 24(6), 1881–1915.
- Shapiro, C. (1983). Premiums for high quality products as returns to reputations. *Quarterly Journal of Economics* 98(4), 659–680.

Appendix A

This Appendix contains the proofs of all results stated in the paper.

Proof of Lemma 1: Suppose $V(0) - V(1) > \frac{m}{\delta\Delta} \Rightarrow V_{mon} > V_{shirk}$. Then the bank always strictly prefers to monitor, so $q(0) = q(1) = 1$. But, substituting $q(0) = q(1) = 1$ in the Bellman equation (10) yields $V(0) - V(1) = 0$, which contradicts incentive compatibility. Therefore, it must be that $V(0) - V(1) = \frac{m}{\delta\Delta}$.

Substituting $\delta\Delta\Lambda = m$ in the Bellman equation (10), and using that to compute the difference $\Lambda = V(0) - V(1)$, we obtain that $\Lambda = (q(0) - q(1))A$. As $\Lambda = \frac{m}{\delta\Delta} > 0$, it must be that $q(0) > q(1)$. ■

Proof of Proposition 1: Substituting $q(0) = 1$ and $q(1) = \hat{q}$, it follows that $\Lambda = V(0) - V(1) = (1 - \hat{q})A$ in a full monitoring equilibrium. Combining this with the incentive compatibility condition, $\Lambda = \frac{m}{\delta\Delta}$, it follows that $\hat{q} = 1 - \frac{m}{\delta\Delta A}$. Substituting $A = \Delta(1 + \beta)(X - C)$ yields the expression for \hat{q} in equation (13). For the equilibrium to be well defined, it must be that $\hat{q} > 0$, which yields the feasibility condition (12) in the Proposition.

We can now solve for the value function $V(d)$. Substituting $q(0) = 1$, $V(0) - V(1) = \frac{m}{\delta\Delta}$, and $V(1) = V(0) - \frac{m}{\delta\Delta}$ in equation (10) yields

$$V(0) = (A - m) + B + \delta \left(V(0) - (1 - p_H) \frac{m}{\delta\Delta} \right) \quad (46)$$

Substituting for A and B from equation (8), and solving for $V(0)$, yields $V(0) = V^*$. Next, since $V(0) - V(1) = \frac{m}{\delta\Delta}$, it must be that $V(1) = V^* - \frac{m}{\delta\Delta}$. ■

Proof of Lemma 2: (1) *Characterizing $E[n_{high}]$.*

As a high reputation bank monitors with probability 1, the probability of default and no default are $1 - p_L - \Delta$ and $p_L + \Delta$, respectively. Therefore, it follows that for any $t \geq 1$, $\Pr(n_{high} = t) = (1 - p_L - \Delta) \cdot (p_L + \Delta)^{t-1}$. Hence,

$$\begin{aligned} E[n_{high}] &= (1 - p_L - \Delta) \cdot \left[\sum_{t=1}^{\infty} t \cdot (p_L + \Delta)^{t-1} \right] \\ &= \frac{1}{1 - p_L - \Delta}, \end{aligned} \quad (47)$$

where the last equation follows by noting that $\sum_{t=1}^{\infty} t \cdot (p_L + \Delta)^{t-1} = \frac{1}{(1-x)^2}$.⁷

(2) As a low reputation bank monitors with probability $p_L + \Delta\hat{q}$, it follows that for any $t \geq 1$, $\Pr(n_{low} = t) = (p_L + \Delta\hat{q}) \cdot (1 - p_L - \Delta\hat{q})^{t-1}$. Hence

$$\begin{aligned} E[n_{low}] &= (p_L + \Delta\hat{q}) \left[\sum_{t=1}^{\infty} t \cdot (1 - p_L - \Delta\hat{q})^{t-1} \right] \\ &= \frac{1}{p_L + \Delta\hat{q}}. \end{aligned} \quad (48)$$

The comparative statics follow by substituting $\hat{q} = 1 - \frac{m}{\delta(1+\beta)\Delta^2(X-C)}$ in the above equation. ■

Proof of Lemma 3: The proof utilizes the following expressions that are obtained by using the Bellman equation (18) in conjunction with the transition equations (15) and (16):

$$V(0) = q(0) \cdot (A - m) + B + \delta(p_L + \Delta q(0)) \cdot (V(0) - V(2)) + \delta V(2), \quad (49a)$$

$$V(1) = q(1) \cdot (A - m) + B + \delta(p_L + \Delta q(1)) \cdot (V(0) - V(2)) + \delta V(2), \quad (49b)$$

$$V(2) = q(1) \cdot (A - m) + B + \delta(p_L + \Delta q(1)) \cdot (V(1) - V(3)) + \delta V(3), \quad (49c)$$

$$\text{and } V(3) = q(2) \cdot (A - m) + B + \delta(p_L + \Delta q(2)) \cdot (V(1) - V(3)) + \delta V(3), \quad (49d)$$

(1) *Proving that $V(0) - V(2) = \frac{m}{\delta\Delta}$.*

We will prove this by contradiction. Suppose $V(0) - V(2) > \frac{m}{\delta\Delta}$. Then, banks with types $x = 0$ and $x = 1$ will strictly prefer to monitor, so that $q(0) = q(1) = 1$, which in turn implies that $V(0) - V(1) = 0$. But if $V(0) = V(1)$, then it must be that $V(1) - V(2) = V(0) - V(2) > \frac{m}{\delta\Delta}$.

Next, subtracting equation (49c) from equation (49b), and using the fact that $V(0) - V(1) = 0$ yields

$$V(1) - V(2) = \delta[1 - p_L - \Delta q(1)] \cdot [V(2) - V(3)] \quad (50)$$

As $V(1) - V(2) > \frac{m}{\delta\Delta} > 0$ and $1 - p_L - \Delta q(1) > 0$, it follows that $V(2) - V(3) > 0$. Combining $V(1) - V(2) > \frac{m}{\delta\Delta}$ and $V(2) - V(3) > 0$ yields that $V(1) - V(3) > \frac{m}{\delta\Delta}$.

Next, if $V(1) - V(3) > \frac{m}{\delta\Delta}$, then it follows that banks with types $x = 2$ and $x = 3$ will strictly prefer to monitor, so that $q(1) = q(2) = 1$. However, $q(1) = q(2)$ implies that $V(2) - V(3) = 0$, which contradicts our earlier finding that $V(2) - V(3) > 0$. Therefore, it must be that $V(0) - V(2) = \frac{m}{\delta\Delta}$. By a similar logic, it can be argued that $V(1) - V(3) = \frac{m}{\delta\Delta}$.

⁷To see why, let $Y = \sum_{t=1}^{\infty} tx^{t-1}$. Then $Y - xY = \sum_{t=0}^{\infty} x^t = \frac{1}{1-x}$, which implies that $Y = \frac{1}{(1-x)^2}$.

(2) *Proving that $q(0) > q(1) > q(2)$.*

After substituting $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$, it is easy to see that $V(0) - V(1) = (q(0) - q(1))A$, and $V(2) - V(3) = (q(1) - q(2))A$. Therefore, it is sufficient to show that $V(0) - V(1) > 0$ and $V(2) - V(3) > 0$.

Note that $V(0) - V(2) = V(1) - V(3)$ implies that $V(0) - V(1) = V(2) - V(3)$. Therefore, it is sufficient to show that $V(2) - V(3) > 0$.

Subtracting equation (49c) from equation (49b), and substituting $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$, yields $V(1) - V(2) = \delta \cdot (V(2) - V(3))$. Therefore,

$$\begin{aligned} V(1) - V(3) &= V(1) - V(2) + V(2) - V(3) \\ &= (1 + \delta) \cdot (V(2) - V(3)) \end{aligned} \tag{51}$$

which proves that $V(2) - V(3) > 0$ because $V(1) - V(3) = \frac{m}{\delta\Delta} > 0$. ■

Proof of Proposition 2: In this proof, we make use of equations (49a) through (49d) that we used in the proof of Lemma 3, after substituting $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$.

Step I: Solving for θ .

Substituting $q(0) = 1$, $q(1) = 1 - \theta$, $q(2) = 1 - 2\theta$, and $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$ in equations (49a) through (49d) that we used in the proof of Lemma 3, it follows that

$$V(0) - V(1) = \theta A, \tag{52}$$

$$V(2) - V(3) = \theta A, \tag{53}$$

and

$$\begin{aligned} V(0) - V(2) &= \theta A + \delta(V(2) - V(3)) \\ &= (1 + \delta)\theta A, \end{aligned} \tag{54}$$

where the second equation above is obtained using equations (52) and (53).

But $V(0) - V(2) = \frac{m}{\delta\Delta}$ by the incentive compatibility constraint (17a). Setting $(1 + \delta)\theta A = \frac{m}{\delta\Delta}$, and solving for θ yields the expression for θ in the proposition. For the equilibrium to be well defined, it must be that $\theta \leq \frac{1}{N} = \frac{1}{2}$, which is equivalent to condition (19).

Step II: Solving the value function $V(x)$ for $x \in \{0, 1, 2, 3\}$.

We begin by solving for $V(0)$. Substituting $q(0) = 1$, $V(0) - V(2) = \frac{m}{\delta\Delta}$, and $\delta V(2) = \delta V(0) - \frac{m}{\Delta}$ in equation (49a) yields $V(0) = A + B + \delta V(0) - \frac{m(1-pL)}{\Delta}$. Substituting for

A and B from equation (8) and solving for $V(0)$ yields $V(0) = V^*$. Once we have solved for $V(0)$, it is fairly straightforward to obtain $V(1)$, $V(2)$ and $V(3)$ using equations (52), (17a), and (17b), respectively. ■

Proof of Lemma 4: (1) We prove part (1) by contradiction. Suppose the inequality in condition (24) is strict for some $d \in \{0, 1\}$. Then, it must be that $q(d) = 1$. Consider the following cases:

Case (a): $\alpha(d) > 0$. In this case, it is possible to choose an alternative $\hat{\alpha} = \alpha(d) - \varepsilon$ where $\varepsilon > 0$ such that the condition (24) still holds. Therefore, $q(d, \hat{\alpha}) = 1$, which means that $P(d, \hat{\alpha}) = p_H(R - C) + C = P(d, \alpha(d))$. But if $P(d, \hat{\alpha}) = P(d, \alpha(d))$, then it must be that $V(d, \hat{\alpha}) > V(d, \alpha(d))$ because the bank places a higher value on immediate liquidity. Hence, in equilibrium, it cannot be that $\alpha(d) > 0$ and condition 24 is strict.

Case (b): Suppose $\alpha(d) = 0$. Then, it must be that $\delta\Delta \cdot (V(0) - V(1)) > m$. Hence, condition (24) holds strictly for both $d = 0$ and $d = 1$, which implies that $q(0) = q(1) = 1$. But then, it follows from the Bellman equation that $V(0) - V(1) = 0$, which contradicts the assumption that $\delta\Delta \cdot (V(0) - V(1)) > m$.

Hence, in equilibrium, the incentive compatibility constraint binds with equality.

(2) If the IC holds with equality, then $\alpha(d)$ is given by equation (28). Moreover, the Bellman equation simplifies to:

$$V(d) = (1 + \beta)(1 - \alpha(d)) \cdot P(q(d)) + \alpha(d) \cdot C - (1 + \beta) + \frac{m p_L}{\Delta} + \delta V(1) \quad (55)$$

Substituting $\alpha C = C - (1 - \alpha)C$ and the expression for $P(q)$ in the above equation yields the expression in equation (29). Computing $V(0)$ and $V(1)$ using the Bellman equation (29), and differencing them yields:

$$\begin{aligned} V(0) - V(1) &= \Delta(1 + \beta)(X - C) \cdot [(1 - \alpha(0)) \cdot q(0) - (1 - \alpha(1)) \cdot q(1)] \\ &\quad + (\alpha(1) - \alpha(0)) \cdot [(1 + \beta)(p_L(X - C) - u) + \beta C] \end{aligned} \quad (56)$$

Suppose $q(0) < q(1)$. Then, it follows from equation (28) that $\alpha(0) > \alpha(1)$. Therefore, since $\Delta(1 + \beta)(X - C) > 0$ and $p_L(X - C) - u > 0$, it follows from the above equation that $V(0) - V(1) < 0$, which must contradict the incentive compatibility condition. Therefore, it must be that $q(0) > q(1)$, which in turn, implies that $\alpha(0) < \alpha(1)$. ■

Proof of Lemma 5: (i) Consider a bank with $d = 1$. It follows from equation (33) and the incentive compatibility condition that $V_{\text{compete}}(1) - \delta V(1) > 0$. Therefore, a bank with

$d = 1$ will always compete with the rival and win the loan.

(ii) Next, consider a bank with $d = 0$. Using equation (33), along with the fact that $\delta\Lambda \geq \frac{m}{\Delta}$, we obtain

$$V_{\text{compete}}(0) - \delta V(0) \leq q(d) \left((1 + \beta) \Delta \cdot \frac{1 - C}{p_L} \right) - \frac{m}{\Delta} (1 - p_L) \quad (57)$$

Condition (35) in the lemma is sufficient to guarantee that $V_{\text{compete}}(0) \leq \delta V(0)$, i.e., that the bank will not compete for the loan this period. ■

Proof of Lemma 6: (1) *Proving that the incentive compatibility condition holds with equality.*

We will prove this by contradiction. Suppose $\Lambda > \frac{m}{\delta\Delta}$. Then, using the same logic as in the proof of Lemma 3, it follows that $q(0) = q(1) = 1$, because banks with $d = 1$ and $d = 0$ will both strictly prefer to monitor.

As $R = R(q)$ when there is no rival, it follows that

$$V(d, \text{none}) = B + q(d) \cdot (A - m + \delta\Delta\Lambda) + \delta(p_L\Lambda + V(1)), \quad (58)$$

where A and B are defined in equation (8).

Therefore, $\Lambda_{\text{none}} \equiv V(0, \text{none}) - V(1, \text{none})$ is given by

$$\begin{aligned} \Lambda_{\text{none}} &= [q(0) - q(1)] \cdot [A - m + \delta\Delta\Lambda] \\ &= 0, \text{ because } q(0) = q(1). \end{aligned} \quad (59)$$

Next, consider the expression for $\Lambda_{\text{rival}} \equiv V(0, \text{rival}) - V(1, \text{rival})$. Here we have to consider the following cases separately:

(i) First, consider the case where a bank with $d = 0$ does not exit when faced with a rival. In this case,

$$\begin{aligned} \Lambda_{\text{rival}} &= [q(0) - q(1)] \cdot \left[(1 + \beta) \Delta \cdot \frac{1 - C}{p_L} - m + \delta\Delta\Lambda \right] \\ &= 0, \text{ because } q(0) = q(1) \end{aligned} \quad (60)$$

But then, equations (59) and (60) together imply that

$$\begin{aligned} \Lambda &= \lambda \Lambda_{\text{rival}} + (1 - \lambda) \Lambda_{\text{none}} \\ &= 0, \end{aligned} \quad (61)$$

which contradicts the assumption that we started with.

(ii) Suppose instead parameter values are such that a bank with $d = 0$ exits when faced with a rival. In this case, $V(0, rival) = \delta V(0)$. We also know from Lemma 5 that $V(1, rival) > \delta V(1)$. Therefore, in this case

$$\begin{aligned}\Lambda_{rival} &< \delta(V(0) - V(1)) \\ &= \delta\Lambda\end{aligned}\tag{62}$$

It follows from (59) and (62) that $\Lambda < \lambda\delta\Lambda$, which contradicts the assumption that $\Lambda > \frac{m}{\delta\Delta} > 0$. Hence, it must be that $\Lambda = \frac{m}{\delta\Delta}$.

(2) *Proving that $q(0) > q(1)$.*

We again prove this by contradiction. Suppose $q(0) \leq q(1)$. Then, by the logic outlined in part (1), it must be that $V(0, none) - V(1, none) \leq 0$ and $V(0, rival) - V(1, rival) < \delta\Delta$, which together imply that $\Lambda < \lambda\delta\Lambda$. But this contradicts the fact that $\Lambda = \frac{m}{\delta\Delta} > 0$. Hence, it must be that $q(0) > q(1)$. \blacksquare

Proof of Proposition 3: (1) *Characterizing the equilibrium when Condition (35) does not hold.*

In this case, after substituting $q(0) = 1$, $q(1) = \hat{q}$ and $\delta\Delta\Lambda = m$, the Bellman equations for $V(d, status)$ (where $status = rival$ or $none$) are as follows:

$$V(0, status) = \delta V(0) + S(R_{status}, 1) - \delta\Lambda(1 - p_L),\tag{63}$$

$$V(1, status) = \delta V(0) + S(R_{status}, \hat{q}) - \delta\Lambda(1 - p_L),\tag{64}$$

where

$$R_{status} = \begin{cases} R(q) & \text{if status=none} \\ R_{rival} & \text{otherwise} \end{cases},\tag{65}$$

$$S(R(q), q(d)) = (1 + \beta) [(p_L + q(d)\Delta)(X - C) + C - 1 - u],\tag{66}$$

and

$$S(R_{rival}, q(d)) = (1 + \beta) q(d)\Delta \cdot \frac{1 - C}{p_L}.\tag{67}$$

(i) Using the above equations, we obtain:

$$\begin{aligned}\Lambda_{rival} &\equiv V(0, rival) - V(1, rival) \\ &= (1 + \beta)(1 - \hat{q})\Delta \cdot \frac{1 - C}{p_L},\end{aligned}\tag{68}$$

and

$$\begin{aligned}\Lambda_{none} &\equiv V(0, none) - V(1, none) \\ &= (1 + \beta)(1 - \hat{q})\Delta(X - C).\end{aligned}\quad (69)$$

Now $\lambda\Lambda_{rival} + (1 - \lambda)\Lambda_{none} = \Lambda = \frac{m}{\delta\Delta}$. Substituting in from (68) and (69), we have

$$\frac{m}{\delta\Delta} = (1 + \beta)(1 - \hat{q})\Delta \left[\lambda \cdot \frac{1 - C}{p_L} + (1 - \lambda)(X - C) \right]. \quad (70)$$

Solving for \hat{q} yields the expression in equation (39). The feasibility condition (38) follows by noting that, for the equilibrium to be well defined, it is necessary that $\hat{q} > 0$.

(ii) Now, $V(0) = \lambda V(0, rival) + (1 - \lambda)V(0, none)$. Substituting for $V(0, rival)$ and $V(1, rival)$ using the Bellman equation (63), and simplifying, we obtain

$$V(0) = \delta V(0) - \delta\Lambda(1 - p_L) + \left(\lambda(1 + \beta)\Delta \cdot \frac{1 - C}{p_L} + (1 - \lambda)v \right) \quad (71)$$

where v is defined in equation (3). Solving the above equation for $V(0)$ yields the expression for $V_{no_exit}^*$ in equation (37). Next, the incentive compatibility condition implies that $V(1) = V(0) - \frac{m}{\delta\Delta} = V_{no_exit}^* - \frac{m}{\delta\Delta}$.

(2) *Characterising the equilibrium when Condition (35) holds.*

The proof is very similar as in part (1), with one exception: $V(0, rival) = \delta V(0)$.

(i) Therefore, in this case,

$$\Lambda_{rival} = -(1 + \beta)\hat{q}\Delta \cdot \frac{1 - C}{p_L} + \frac{m(1 - p_L)}{\Delta}, \quad (72)$$

where we have again made use of $\delta\Lambda = \frac{m}{\Delta}$. The expression for Λ_{none} is still given by (69).

As before, $\lambda\Lambda_{rival} + (1 - \lambda)\Lambda_{none} = \Lambda = \frac{m}{\delta\Delta}$. Substituting from (69) and (72), we have

$$\frac{m}{\delta\Delta} = \lambda \left[\frac{m(1 - p_L)}{\Delta} - (1 + \beta)\hat{q}\Delta \cdot \frac{1 - C}{p_L} \right] + (1 - \lambda)(1 + \beta)(1 - \hat{q})\Delta(X - C). \quad (73)$$

Solving for \hat{q} yields the expression in equation (41). The feasibility condition (40) follows by noting that the equilibrium is well defined only if $\hat{q} > 0$.

(ii) In this case, by a similar logic as in Step 1(i), it follows that

$$V(0) = \delta V(0) + (1 - \lambda) \left[v - \frac{m(1 - p_L)}{\Delta} \right] \quad (74)$$

Solving the above equation for $V(0)$ yields $V(0) = (1 - \lambda)V^*$, where V^* is defined in equation (11). The incentive compatibility condition implies that $V(1) = V(0) - \frac{m}{\delta\Delta} = (1 - \lambda)V^* - \frac{m}{\delta\Delta}$. ■

Proof of Proposition 4: A bank with reputation d will choose to lend γ instead of 1 if, and only if, $V_\gamma \geq V(d)$. For $d = 1$ (low-reputation bank), this condition is equivalent to

$$\begin{aligned} S_\gamma + \delta \left[V^* - (1 - p_L) \frac{m}{\delta\Delta} \right] &\geq V^* - \frac{m}{\delta\Delta} \\ \text{i.e., } S_\gamma &\geq (1 - \delta)V^* + \delta(1 - p_L) \frac{m}{\delta\Delta} - \frac{m}{\delta\Delta} \end{aligned} \quad (75)$$

Substituting for V^* and S_γ from Eqns. (11) and (44), respectively, and rearranging yields the following condition

$$\gamma \geq 1 + \frac{\Delta(X - C) - \frac{m}{\delta\Delta(1+\beta)}}{[p_L(X - C) + C - 1 - u]} \equiv \gamma_l. \quad (76)$$

Clearly, $\gamma_l > 1$ if the feasibility condition (12) is met. It is easily verified that γ_l is increasing in Δ , $(X - C)$, δ and β , and is decreasing in p_L and m .

By a similar logic, it can be shown that a high-reputation bank ($d = 0$) will choose to lend γ instead of 1 if, and only if,

$$\gamma \geq 1 + \frac{\Delta(X - C)}{p_L(X - C) + C - 1 - u} \equiv \gamma_h. \quad (77)$$

Proposition 4 follows by noting that $\gamma_h > \gamma_l$. ■

Appendix B

7 B1. Alternative setup to model competition

In Section 4, we modeled the effect of competition on the incumbent bank's monitoring incentives by assuming that the rival lender was short-lived, and hence, had no incentives to monitor. Effectively, this lowers the loan's repayment value to R_{rival} if a rival lender appears.

Suppose we modify our base model of competition as follows: As before, in each period, there is a probability λ that a rival bank will compete with the incumbent bank for the period's borrower. Unlike before, the rival itself is assumed to have a reputation. It has a high reputation with probability ρ and a low reputation with probability $1 - \rho$. Thus, the incumbent bank's scenarios are as follows: no competition with probability $1 - \lambda$, competition from a low-reputation bank with probability $\lambda(1 - \rho)$, and competition from a high-reputation bank with probability $\lambda\rho$.

As before, let $V(d, none)$ denote the bank's expected discounted profits when no rival is present. Let $V(d, rival_0)$ and $V(d, rival_1)$ denote expected discounted profits when faced with rival with reputation $d = 0$ and $d = 1$, respectively. We use \hat{d} to denote the rival's reputation, and d to denote the incumbent's reputation. So the expected value of having a reputation d is

$$V(d) = \lambda\rho V(d, rival_0) + \lambda(1 - \rho) V(d, rival_1) + (1 - \lambda) V(d, none) \quad (78)$$

Incentive compatibility would then require $\Lambda \geq \frac{m}{\delta\Delta}$. Let's assume for now (to be proved formally later) that $\Lambda = \frac{m}{\delta\Delta}$ and $q(0) > q(1)$.

Let $R_{be}(d)$ denote the lowest loan repayment value at which a bank with reputation d breaks even. $R_{be}(d)$ must satisfy the following equation

$$(p_L + q(d)\Delta)(R_{be}(d) - C) + C = 1. \quad (79)$$

As $q(0) > q(1)$, it must be that $R_{be}(0) < R_{be}(1)$. Therefore, if a rival appears, the loan's repayment value is set by the bank with the worse reputation, i.e., by $d_{\max} \equiv \max\{d, \hat{d}\}$. Now, $d_{\max} = 1$ unless $d = \hat{d} = 0$ (i.e., unless both the incumbent and the rival have a high reputation). Therefore, the loan's repayment value when a rival appears is $R_{be}(1)$, unless both the rival and the incumbent are in the high reputation state, in which case, it is $R_{be}(0)$. On the other hand, if a rival does not appear, then the repayment value of the loan is $R(q(d))$, as given by equation (1).

In the event of competition, the incumbent (rival) may either compete with the rival (incumbent) for that period's loan or cede the loan to the rival (incumbent). If it decides to compete, then its current period surplus is

$$\begin{aligned} S(d, R_{be}(d_{\max})) &= (1 + \beta) \{ (p_L + q(d) \Delta) (R_{be}(d_{\max}) - C) + C - 1 \} \\ &= \frac{(1 + \beta) (1 - C) [q(d) - q(d_{\max})] \Delta}{(p_L + q(d_{\max}) \Delta)}, \end{aligned} \quad (80)$$

where the second equation is obtained after substituting $R_{be}(d) - C = \frac{1-C}{p_L+q(d)\Delta}$ (from equation 79). The expected value of its discounted profits if it chooses to compete is

$$\begin{aligned} V_{\text{compete}}(d, \hat{d}) &= S(R_{be}(d_{\max})) - mq(d) + \delta q(d) \cdot \Delta \Lambda + \delta p_L \Lambda + \delta V(1) \\ &= S(R_{be}(d_{\max})) + \frac{mp_L}{\Delta} + \delta V(1), \end{aligned} \quad (81)$$

where the second equation follows by noting that $\Lambda = \frac{m}{\delta \Delta}$.

Instead, if it chooses not to compete, then its expected value is $\delta V(d)$. Therefore,

$$V(d, rival_{\hat{d}}) = \max \left\{ \delta V(d), V_{\text{compete}}(d, \hat{d}) \right\}. \quad (82)$$

Our next result characterizes the behavior of the incumbent and the rival in the event of competition.

Lemma 7 *In the event there is competition:*

1. *A bank in the low reputation state ($d = 1$) will always compete for the loan.*
2. *A bank in the high reputation state ($d = 0$) will not compete with another bank in the high reputation state. It will compete with a bank in the low reputation state if, and only if,*

$$\frac{(1 + \beta) (1 - C) (q(0) - q(1)) \Delta}{(p_L + q(1) \Delta)} > \frac{m(1 - p_L)}{\Delta}. \quad (83)$$

Proof of Lemma 7: (1) Consider a bank with $d = 1$ faced with a rival. Then, the loan's repayment is $R_{be}(1)$. By the definition of $R_{be}(d)$ in equation (79), it follows that $S(1, R_{be}(1)) = 0$. Therefore, regardless of \hat{d}

$$V_{\text{compete}}(1, \hat{d}) = \delta p_L \Lambda + \delta V(1).$$

As $\Lambda = \frac{m}{\delta \Delta} > 0$, it follows that $V_{\text{compete}}(1, \hat{d}) > \delta V(1)$; i.e., a bank with $d = 1$ will always compete with the rival. By symmetry, it follows that a rival with $d = 1$ will always compete with the incumbent.

(2) Next consider a bank with $d = 0$. Here we need to consider the following cases separately because the loan repayment value varies depending on \hat{d} .

(a) Competition from a high reputation rival ($\hat{d} = 0$): In this case, the loan repayment value is $R_{be}(0)$, and the current surplus from competing is $S(0, R_{be}(0)) = 0$ (by the definition of $R_{be}(0)$). Therefore,

$$V_{\text{compete}}(0, 0) = \delta p_L \Lambda + \delta V(1).$$

As $V_{\text{compete}}(0, 0) < \delta \Lambda + \delta V(1) = \delta V(0)$, a high reputation bank will not compete with another high reputation bank.

(b) Competition with a low reputation rival ($\hat{d} = 1$): In this case, the loan repayment value is $R_{be}(1)$, and the incumbent's current period surplus if it competes is $S(0, R_{be}(1)) > 0$. Hence, the incumbent will compete if, and only if, $S(0, R_{be}(1)) + \delta p_L \Lambda + \delta V(1) > \delta V(0)$. Substituting for $S(0, R_{be}(1))$ from equation (80), and simplifying, yields condition (83) in the Lemma. ■

Note that Lemma 7 is consistent with Lemma 5 in the paper, and confirms that the high reputation bank may exit even when faced with a low reputation rival, whereas the low reputation bank will always compete for the loan regardless of the rival's reputation because it has nothing to lose.

B2. Alternative setup to model lending booms

In Section 5 where we analyzed the bank's monitoring incentives during lending booms, we made a simplifying assumption that the market participants could not observe the bank's lending volume during the boom period. In this section, we relax this restriction, and assume that market participants observe the bank's lending volume in addition to its loan performance. Thus, the bank's reputation in the period following a lending boom could depend both on its volume of lending and loan performance during the boom period, and can take on four possible values: 0, 1, 0_γ and 1_γ , where 0_γ and 1_γ denote no default and default, respectively for a bank that lent γ during the boom period. For tractability, we assume that lending booms can occur at most once in every n periods, where $n \gg 1$. What this assumption does is to return the bank to one of two reputation states, $d = 0$ or $d = 1$, in the second period following a lending boom. We also assume that while participants can observe the bank's lending volume, they do not observe the bank's loan demand; i.e., participants cannot differentiate between a bank that faced a demand of 1 and another that faced a demand of γ but chose to lend 1. Thus, the bank cannot enhance its reputation simply by choosing the low lending volume in a lending boom.

Consider the behavior of bank with reputation $d \in \{0, 1\}$ in a boom period. If it lends γ , then it will choose its monitoring to

$$\max_q -\gamma m q + \delta [(p_L + q\Delta) \cdot (V(0_\gamma) - V(1_\gamma)) + V(1_\gamma)],$$

where $V_\gamma(0)$ and $V_\gamma(1)$ denote the value of having reputation 0_γ and 1_γ , respectively

Hence, a bank that lends γ will monitor if, and only if,

$$\begin{aligned} \delta\Delta \cdot (V(0_\gamma) - V(1_\gamma)) &\geq \gamma m \\ \text{i.e., } V(0_\gamma) - V(1_\gamma) &\geq \gamma \cdot (V(0) - V(1)), \end{aligned} \quad (84)$$

because $\delta\Delta(V(0) - V(1)) = m$.

Next, let us try to characterize $V(0_\gamma)$ and $V(1_\gamma)$. Let t denote the boom period. Using the fact that the bank will have reputation of either $d = 0$ or $d = 1$ in time $t + 2$ (because the next lending boom doesn't occur for a while), the Bellman equations for banks with reputations 0_γ and 1_γ in period $t + 1$ will be as follows:

$$\begin{aligned} V(0_\gamma) &= q(0_\gamma) \cdot A + B - mq(0_\gamma) + \delta [(p_L + q(0_\gamma) \cdot \Delta)(V(0) - V(1)) + V(1)] \\ &= q(0_\gamma) \cdot A + B + \frac{mp_L}{\Delta} + \delta V(1), \end{aligned} \quad (85)$$

because $\delta\Delta(V(0) - V(1)) = m$, and

$$V(1_\gamma) = q(1_\gamma) \cdot A + B + \frac{mp_L}{\Delta} + \delta V(1). \quad (86)$$

Therefore, $V(0_\gamma) - V(1_\gamma) = (q(0_\gamma) - q(1_\gamma)) \cdot A$.

Lemma 8 *In any equilibrium where a bank that lends γ during a lending boom also monitors, it must be that $V(0_\gamma) - V(1_\gamma) = \gamma \cdot (V(0) - V(1))$. Such an equilibrium exists only if*

$$\gamma m < \delta(1 + \beta)\Delta^2(X - C). \quad (87)$$

Proof of Lemma 8: If condition (84) holds with strict inequality, then $q(0_\gamma) = q(1_\gamma) = 1$, which implies that $V(0_\gamma) - V(1_\gamma) = 0$; contradiction. Therefore, it must be that condition (84) holds with equality, which implies that

$$q(0_\gamma) - q(1_\gamma) = \frac{\gamma m}{A}. \quad (88)$$

But then,

$$q(1_\gamma) = q(0_\gamma) - \frac{\gamma m}{A} \leq 1 - \frac{\gamma m}{A}.$$

For the equilibrium to be well defined, it must be that $q(1_\gamma) > 0$, which in turn, requires that $\gamma m < A$. Substituting for A yields condition (87). ■

Let us focus on the more interesting case where γ is large enough that condition (87) is violated. Then, regardless of its reputation, a bank that lends γ during the boom period will not monitor the loan. Hence, the current period surplus from lending γ is

$$S_\gamma = \gamma(1 + \beta) \cdot [p_L(X - C) + C - 1 - u], \quad (89)$$

and the total expected value of lending γ during the boom period (which also doesn't depend on reputation d) is

$$V_\gamma = S_\gamma + \delta [p_L \cdot (V(0_\gamma) - V(1_\gamma)) + V(1_\gamma)]. \quad (90)$$

Instead, if the bank lends 1 during the boom period, its expected value is $V(d)$. Hence, the bank will increase its lending volume to d if, and only if, $V_\gamma \geq V(d)$.

We are interested in mechanisms that maximize the intensity of monitoring. So let us consider the full monitoring equilibrium characterized in Proposition 1; $q(0) = 1$, $q(1) = \hat{q}$, $V(0) = V^*$ and $V(1) = V^* - \frac{m}{\delta\Delta}$. We still need to characterize $q(0_\gamma) = q(1_\gamma) = 0$. Given that a bank that lends γ does not monitor, any mechanism that seeks to encourage monitoring must penalize the bank for lending γ in the boom period. This can be achieved by setting $q(0_\gamma) = q(1_\gamma) = 0$, such that

$$V(0_\gamma) = V(1_\gamma) = V^* - \Delta(1 + \beta)(X - C). \quad (91)$$

Substituting for $V(0_\gamma)$ and $V(1_\gamma)$ in the expression for $V(d, \gamma)$ yields

$$V(d, \gamma) = S_\gamma + \delta [V^* - \Delta(1 + \beta)(X - C)]. \quad (92)$$

Hence, a bank with reputation d will prefer to lend γ in the boom period only if

$$S_\gamma + \delta [V^* - \Delta(1 + \beta)(X - C)] \geq V(d), \quad (93)$$

As $V(0) = V^*$ and $V(1) = V^* - \frac{m}{\delta\Delta}$, the γ threshold above which condition (93) is met will be higher for the bank with $d = 0$. Thus, the result in Proposition 4 should still hold.