Dealer Attention, Liquidity Spillovers, and Endogenous Market Segmentation *

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Abstract

We describe a new mechanism that explains the transmission of liquidity shocks from one security to another ("liquidity spillovers"). We consider a model in which two securities are traded by two different pools of risk averse dealers. The payoffs of these securities are correlated. Hence, dealers in one security can learn information from the price of the other security. As securities' prices are noisier when markets are less liquid, a decline in liquidity in one market spreads to the other market. This spillover mechanism relies on dealer attention to the price of other securities. Thus, we also analyze how the cost of attention affects market liquidity. Interestingly, a reduction in the cost of attention does not necessarily improve liquidity if too few dealers pay this cost. Moreover, for some parameter values, attention decisions to prices by different dealers are complements. Thus, multiple equilibria with varying levels of attention and liquidity can emerge for the same values of the fundamentals.

Keywords: Commonality in liquidity, Limited Attention, Value of price information, Liquidity externalities, Transparency, High frequency market-making.

JEL Classification Numbers: G10, G12, G14

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1 Introduction

Various studies document the existence of co-movements in liquidity across securities (e.g., Chordia et al. (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001) for stocks and Chordia et al. (2005) for bonds and stocks). This phenomenon has important implications for asset pricing (see Amihud et al. (2005) for a survey) but its causes are not well understood. One possibility is that financing constraints bind liquidity providers in different securities at the same time and constitute thereby a systematic liquidity factor. This mechanism is formalized by Gromb and Vayanos (2002) and Brunnemeier and Pedersen (2007) and has received empirical support for NYSE stocks (see for instance, Coughenour and Saad (2004) or Comerton-Forde et al. (2010)).

In this paper we describe another mechanism that generates co-variations in liquidity. Essential to this mechanism is the fact that dealers in a security often rely on the prices of other securities to set their quotes. For instance, dealers in a stock learn information from the prices of its industry peers or stock index futures, dealers in a CDS learn information from the underlying stock price, dealers of ADRs learn information from the issuing firm's domestic stock price, etc...

Learning from the price of other securities is a source that generates *liquidity spillovers.* Consider a dealer in security X who monitors price movements in security Y. These movements can be due to (uninformative) demand shocks or news about fundamentals. Now suppose that a shock *specific* to security Y decreases the cost of liquidity provision for dealers in this security (e.g., dealers in this security face less stringent limits on their positions). Then the price of security Y becomes more informative for dealers in security X since demand shocks in security Y contribute less to its volatility relative to news about fundamentals. Thus, inventory risk for dealers in security X is lower and the cost of liquidity provision for these dealers declines as well. In other words, the improvement in liquidity for security Y spreads to security X (see Figure 1).

To formalize this intuition, we consider a model with distinct pools of risk averse dealers operating in two different securities, say X and Y, with a two-factor structure. Each pool has perfect information on one of the two factors but not on both and the two pools of dealers have information on different factors. Depending on the securities' factor loadings, learning can be *two-sided* (dealers in each security learn from each other's price) or *one-sided* (e.g., the price of X is informative for dealers in security Y but not vice versa).[1] We call dealers who engage in cross-security price monitoring "pricewatchers." The fraction of pricewatchers in a given security sets *its dealers' level of attention* to the other security.

The model delivers the spillover mechanism that we described earlier and additional findings.[2] First, when learning is two-sided, an exogenous shock to the cost of liquidity provision

[1] For instance, dealers in a stock can learn information on common factors driving stock prices from the price of a stock index futures whereas dealers in the futures market are unlikely to learn information from individual stocks. In this case, learning is one-sided.

[2] Iliquidity in our model is measured by the sensitivity of prices to demand shocks. The market is more liquid when this sensitivity is low. This is consistent with, for instance, Kyle (1985).

in one security (say Y) is amplified by the propagation of this shock to the cost of liquidity provision in the other security (say X). Indeed, as learning is two-sided, the change in the liquidity of security X feeds back on the liquidity of security Y, which sparks a chain reaction amplifying the initial shock.

Second, when learning is two-sided, the model can feature multiple equilibria with differing levels of liquidity. The reason is as follows. Suppose that dealers in security X *expect* a drop in the liquidity of security Y. Then, dealers in security X expect the price of security Y to be noisier, which makes the market for security X less liquid. But as a consequence, the price of security X becomes less informative for dealers in security Y and the liquidity of security Y drops, which validates the expectation of dealers in security X. Hence, dealers' expectations about the liquidity of the other security can be *self-fulfilling*. For this reason, there exist cases in which, for the same parameter values, the liquidity of securities X and Y can be either relatively high or relatively low.[3] A sudden switch from a high to a low liquidity equilibrium is an extreme form of co-variation in liquidity since it corresponds to a situation in which the liquidity of several related securities becomes low at once, without an apparent reason (see Figure 1).

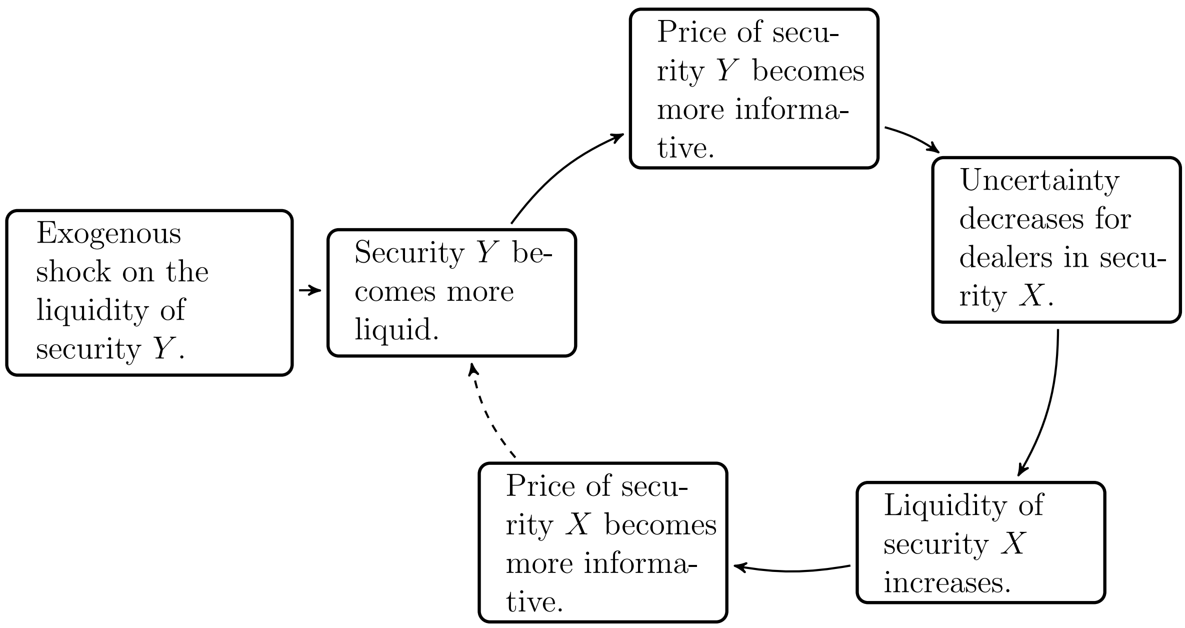


Figure 1: Information driven liquidity spillover.

Third, an increase in the fraction of pricewatchers has an ambiguous impact on liquidity. On the one hand, this increase improves liquidity because pricewatchers require a smaller compensation for inventory risk (as they have more information). On the other hand, entry of new pricewatchers impairs liquidity because it raises adverse selection risk for remaining dealers. Indeed, these dealers are more likely to end up with a long position when the asset value is low and a short position when the asset value is high. In anticipation of this form of winner's curse, they demand a greater compensation to provide liquidity. The net effect on

[3] There also exist cases in which the equilibrium is unique, even if learning is two-sided.

liquidity is always positive when dealers' risk bearing capacity (i.e., the variance of dealers' aggregate dollar inventory divided by dealers' risk tolerance) is sufficiently high. Otherwise, liquidity may drop when the market features more pricewatchers.

In a last step, we endogenize the fraction of pricewatchers by introducing a cost of attention to prices. There are several possible interpretations for this cost. In the absence of real time price reporting (as for instance in some OTC markets), real time price information is available only to a few privileged dealers and very costly to collect for other participants.[4] More generally, data vendors (Reuters, Bloomberg, etc...) or trading platforms charge a fee for real time datafeed.[5] Last, in fast markets, prices are changing quickly and tracking price changes requires attention. Automation of market-making ("algorithmic trading") relaxes attention constraints but it requires investments (computers, co-location fees, etc...).

When learning is one-sided, the value of price monitoring declines with the fraction of pricewatchers. Thus, the equilibrium fraction of pricewatchers is unique and inversely related to the cost of monitoring. When dealers' risk bearing capacity is large, a decrease in the cost of monitoring leads to an improvement in liquidity. Otherwise, liquidity is a U-shaped function of the cost of monitoring. Indeed, for relatively high values of the cost of monitoring, a decrease in this cost triggers entry of a few pricewatchers, which creates adverse selection risk for non pricewatchers and impairs liquidity.

In contrast, when liquidity spillovers are two-sided, the value of monitoring the price of, say, security X for dealers in security Y can *increase* with the fraction of pricewatchers in security Y (for some parameter values). The reason is as follows. As explained previously, if dealers' risk bearing capacity is low enough, an increase in the fraction of pricewatchers in security Y makes this security more liquid. This improvement in liquidity then spreads to security X, which makes the price of this security more informative. Thus, monitoring security X becomes more valuable for dealers in security Y. A similar, albeit more direct, mechanism implies that the value of monitoring the price of security X for dealers in security Y also increases in the fraction of pricewatchers in security X. Indeed, as the number of pricewatchers in security X increases, the price of this security becomes more informative, which strengthens its value for dealers in security Y.

In summary, when spillovers are two-sided, the value of monitoring the price of another security for dealers in one security can increase in the number of pricewatchers in either security. This finding is surprising since usually the value of financial information declines with the number of investors buying information (Grossman and Stiglitz (1980) or Admati and Pfleiderer (1986)). Our model reveals that this principle does not necessarily apply to price information, the reason being that the precision of price information increases in the number of dealers

[4]For instance, a dealer in a bond may be an employee of a trading firm also active in the CDS market. In this way, the dealer may be privy of information on trades in the CDS market not available to his competitors.

[5]Market participants often complain about these data fees.For instance, the fee charged by Nasdaq for the dissemination of corporate bond prices has been very controversial. For accounts of these debates, see, for instance, "Latest Market Data Dispute Over NYSE's Plan to Charge for Depth-of-Book Data Pits NSX Against Other U.S. Exchanges," Wall Street Technology, May 21, 2007; the letter to the SEC of the Securities Industry and Financial Markets Association (SIFMA) available at `http://www.sifma.org/regulatory/comment_letters/41907041.pdf`, and "TRACE Market Data Fees go to SEC," Securities Industry News, 6/3/2002.

buying this information.

For this reason, for a given cost of price information, the number of pricewatchers in a security can be high or low. That is, *for identical parameter values*, the markets for the two securities can appear relatively well integrated (the fraction of pricewatchers is high) or segmented (the level of pricewatchers is low). As an illustration we construct an example in which, for a fixed correlation in the payoffs of both securities, the markets for securities X and Y are either fully integrated (all dealers are pricewatchers) or segmented (no dealer is a pricewatcher). For dealers in security X, monitoring the price of the other security does not have much value if there are no pricewatchers in security Y and vice versa. Thus, the situation in which the two markets are segmented is self-sustaining and can persist even if the cost of attention declines.

This result has interesting implications. First, it implies that fads, traditions, or other coordination devices may determine the degree of integration between two securities, independently of the correlation in the payoffs of these securities. Second, a decrease in the cost of attention (due for instance to better information linkages between markets) does not per se entail greater market integration, unless the cost is very low. Third, dealers operating in related but opaque segments may undervalue the benefit of greater market integration. Indeed, in the low attention equilibrium, the value of getting price information is low. Thus, data vendors will perceive a weak demand and will therefore lack incentives to collect and disseminate price information. In this case, regulatory intervention is needed. A case in point is the U.S. corporate bond market where real time dissemination of bond prices took off only under regulatory pressure (see Bessembinder et al. (2006)).

The rest of the paper is organized as follows. Section 2 provides a brief review of the literature related to our paper. Section 3 describes the model. In Section 4, we consider the case in which the fraction of pricewatchers is fixed. We show how liquidity spillovers and multiple equilibria arise in this set-up. In Section 5, we study how the value of price information depends on the fraction of pricewatchers and we endogenize this fraction. Section 6 discusses testable implications of the model and Section 7 concludes. Proofs are collected in the Appendix.

2 Literature Review

Our paper is related to several strands of research. First, it contributes to the literature on co-movements in liquidity. As explained previously, the literature emphasizes the role of market-wide variations in dealers' financing constraints as a source of liquidity comovements. Our model describes another mechanism based on cross asset price monitoring. This mechanism does not rule out a role for financing constraints. In fact, funding restrictions for dealers in one asset class (e.g., stocks) can be the spark that triggers a drop in the liquidity of this class of assets. But, in contrast to the extant literature, our model predicts that this shock can spread to other asset classes (e.g., bonds) even if there is no tightening of funding constraints for dealers in other asset classes. The only requirement is that the prices of assets in the first class are used as a source of information to price assets in other classes.

Second, our paper relates to the growing literature relating market liquidity to attention constraints. Duffie (2010) argues that the immediate price impact of supply shocks and the subsequent price dynamics can reflect "...many sorts of attention costs." Foucault, Röell and Sandas (2003) studies the impact of imperfect monitoring on adverse selection and Biais, Hombert and Weill (2010) analyzes price dynamics in limit order markets in the presence of limited attention. Recent empirical papers (Corwin and Coughenour (2008), Boulatov et al. (2010) and Chakrabarty and Moulton (2009)) find that attention constraints for NYSE specialists have an effect on market liquidity. Thus, modelling dealer attention is important to understand liquidity. Our paper is an effort in this direction.

Next, our paper relates to the literature on multi-market trading (e.g., Chowdry and Nanda (1991)) and cross asset price pressures (e.g., Pasquariello (2007), Andrade, Chang and Seasholes (2008), Bernhardt and Taub (2008), Pasquariello and Vega (2009), Boulatov, Hendershott and Livdan (2010)). Chowdry and Nanda (1991) focuses on the case in which the same security trades in two different markets and dealers in one market cannot condition their price on the price in the other market. Other papers (e.g., Bernhardt and Taub (2008) or Pasquariello and Vega (2009)) consider models with multiple assets and assume that dealers in one security can condition their price on the prices of all other securities.[6] Our assumptions differ in many ways. For instance, we consider risk averse dealers while these models consider risk neutral dealers. Moreover we consider the effect of varying the level of attention of dealers to the price in another market and we endogenize this level. Thus, we can analyze how market liquidity and market integration depend on the cost of attention.

Our set-up is closer to King and Wadhwani (1990) who study volatility spillovers across markets. They analyze how news in one market are transmitted to another market and they explicitly assume that prices are the conduit through which information is transmitted. However, they do not relate the informativeness of the price in one market to the liquidity of this market, as we do. We show that this relationship is a source of liquidity spillovers and can lead to multiple equilibria with different levels of price informativeness and liquidity, a possibility which does not arise in King and Wadhwani (1990). It is well-known that participation externalities result in multiple equilibria with differing levels of liquidity (see Admati and Pfleiderer (1988), Pagano (1989), and Dow (2004) for example). However, to the best of our knowledge, the coordination problems that arise in our model, which involves dealers operating in different securities, have not been analyzed before in the literature.

Last, our paper is related to the literature on the value of financial information (e.g., Grossman and Stiglitz (1980), Admati and Pfleiderer (1986)). We indirectly contribute to this literature by studying the value of securities price information. As explained previously, we show that price information is special in the sense that its value can increase with the number of investors buying this information, an effect which does not arise in standard models of information acquisition. In this respect, our paper adds to the few papers identifying conditions under which the value of financial information may increase with the number of informed investors

[6]Bernhardt and Taub (2008) compare equilibrium outcomes when informed investors can condition their market orders in one security on the prices of all other securities and when they cannot.

(Barlevy and Veronesi (2000), Veldkamp (2006), and Chamley (2007)).

3 The model

We consider two securities, D and F, traded by two distinct pools of traders. The payoffs of these securities, v_D and v_F, are given by a factor model with two common risk factors δ_D and δ_F, i.e.,

$$v_D = \delta_D + d_D \times \delta_F + \eta, \tag{3.1}$$

$$v_F = d_F \times \delta_D + \delta_F + \nu. \tag{3.2}$$

The random variables δ_D, δ_F, η and ν are independent and have a normal distribution, with mean zero. The variance of η is denoted σ_η^2. We make additional parametric assumptions that simplify the exposition without affecting our conclusions. First, there is no idiosyncratic risk for security F (i.e., $\nu = 0$). Second, the variance of the factors is normalized to one. Third, we assume that $d_F = 1$ and $d_D \in [0, 1]$, so that the payoffs of the two securities are positively correlated. To simplify notations, we will therefore denote d_D simply by d, unless a confusion is possible. When $d = 0$, the payoff of security D does not depend on factor δ_F. Thus, the price of security F cannot convey new information to dealers in security D. In this case, we say that learning is one-sided.

In each market, there are two types of traders: *(i)* a continuum of risk-averse speculators and *(ii)* liquidity traders. The aggregate demand of liquidity traders in market j is $u_j \sim N(0, \sigma_{u_j}^2)$. Liquidity demands in both markets are independent. The net order imbalance from liquidity traders is absorbed by speculators. Hence, in the rest of the paper, we refer to speculators as *dealers* and to u_j as the size of the *demand shock* in market j.

Dealers are specialized: they are active in only one security. Dealers specialized in security j have perfect information on factor δ_j but no information on factor δ_{-j}. However, they can follow the price of the other security to obtain information on factor δ_{-j}. We call μ_j, the level of attention to security $-j$ and we denote by μ_j the fraction of dealers specialized in security j who monitor the price of security $-j$. We refer to these dealers as being "insiders" or *pricewatchers.* Other dealers are called outsiders. We index decisions of insiders by I and decisions of outsiders by O. The polar cases in which there are no pricewatchers in either market ($\mu_D = \mu_F = 0$) on the one hand and all dealers are pricewatchers ($\mu_D = \mu_F = 1$) on the other hand are called the "no attention case" and the "full attention case," respectively. Table 1 summarizes the various possible cases that will be considered in the paper.

Each dealer in market j has a CARA utility function with risk tolerance γ_j. Thus, if dealer i in market j holds x_{ij} shares of the risky security, her expected utility is

$$E\left[U\left(\pi_{ij}\right)|\delta_j, \Omega_j^k\right] = E\left[-\exp\left\{-\gamma_j^{-1}\pi_{ij}\right\}|\delta_j, \Omega_j^k\right], \tag{3.3}$$

where $\pi_{ij} = (v - p_j)x_{ij}$ and Ω_j^k is the *price information* available to a dealer with type $k \in \{I, O\}$ operating in security j.

Attention/Learning	One-Sided: $d = 0$	Two-Sided: $d > 0$
No Attention	$\mu_D = \mu_F = 0$	$\mu_D = \mu_F = 0$
Limited Attention	$\mu_j > 0$ and $\mu_{-j} < 1$	$\mu_j > 0$ and $\mu_{-j} < 1$
Full Attention	$\mu_D = \mu_F = 1$	$\mu_D = \mu_F = 1$

Table 1: Various Cases

As dealers submit price contingent demand functions, they all act as if they were observing the clearing price in their market. Thus, we have $\Omega_j^I = \{p_j, p_{-j}\}$ and $\Omega_j^O = \{p_j\}$. We denote the demand function of an insider by $x_j^I(\delta_j, p_j, p_{-j})$ and that of an outsider by $x_j^O(\delta_j, p_j)$. In each period, the clearing price in security j, p_j, is such that the demand for this security is equal to its supply, i.e.,

$$\int_0^{\mu_j} x_j^I(\delta_j, p_j, p_{-j})di + \int_{\mu_j}^1 x_j^O(\delta_j, p_j)di + u_j = 0, \quad \text{for } j \in \{D, F\}. \tag{3.4}$$

As in many other papers (e.g., Kyle (1985) or Vives (1995)), we will measure the level of illiquity in security j by the sensitivity of the clearing price to the demand shock.

There are several ways to interpret the two securities in our model. For instance, as in King and Wadhwani (1990), securities D and F could be two stock market indexes for two different countries. Alternatively, they could represent a derivative and its underlying security. For instance, security D could be a credit default swap (CDS) and security F the stock of the firm on which the CDS is written.[7] When $d = f = 1$ and $\sigma_\eta^2 = 0$, the payoff of the two securities is identical, as in Chowdry and Nanda (1991). In this case, the two securities can be viewed as the stock of a cross-listed firm and its ADR in the foreign market. Factor δ_j can then be viewed as the component of the firm's cash-flows that comes from its sales in country j by the cross-listed firm. In each of these cases, it is natural to assume that dealers have specialized information. For instance, dealers in country j will be well informed on local fundamental news but not on foreign fundamental news as in King and Wadhwani (1990).[8]

4 Learning from prices and liquidity co-movements

4.1 Benchmark: the no attention case

We first analyze the equilibrium in the no attention case ($\mu_D = \mu_F = 0$). For instance, the markets for securities D and F may be opaque so that dealers in each security can obtain information on the price of the other security only after some delay. Alternatively, the prices of each security are available in real time but accessing this information is costly for dealers.

[7] Dealers in CDS are often making the market in the bonds on which CDS are written. In contrast, dealers in equity markets and CDS are different.

[8] In the case of the CDS market, dealers in CDS are often affiliated with lenders and therefore better informed on the likelihood of defaults (and size of associated losses) than dealers in the stock market (see Acharya and Johnson (2007))

If the cost is high enough, no dealer chooses to be informed on the price of the other security (see Section 5).

Lemma 1. *When $\mu_F = \mu_D = 0$, the equilibrium price in market j is:*

$$p_j = \delta_j + B_{j0} u_j, \tag{4.1}$$

with $B_{D0} = \gamma_D^{-1}(\sigma_\eta^2 + d^2)$ and $B_{F0} = \gamma_F^{-1}$.

Coefficient B_{j0} measures the sensitivity of the equilibrium price for security j to the aggregate demand shock in this market (we use index "0" to refer to the case in which $\mu_F = \mu_D = 0$). It is therefore a measure of illiquidity for this security. As usual, the illiquidity of a security increases if dealers are more risk-averse (γ_j decreases) or more uncertain about the value of the security (e.g., σ_η^2 increases).

In the no attention case, parameters $\{\sigma_\eta^2,\ d,\ \gamma_D\}$ only affect the illiquidity of security D. Hence, we refer to these parameters as being the "liquidity fundamentals" of security D. Similarly, we refer to γ_F as a liquidity fundamental of security F since it does not affect the illiquidity of security D. In the benchmark case, there are no liquidity spillovers: a change in the illiquidity of one market (due to a change in one of its illiquidity fundamental) does not affect the illiquidity of the other market. For instance, an increase in the risk tolerance of dealers in security D makes this security more liquid but has no effect on the illiquidity of the other security. In contrast, with limited or full attention, the shock to the liquidity fundamental of one security will affect the liquidity of the other security, as shown in the next sections.

4.2 Full attention generates positive liquidity spillovers

In this section, we consider the polar case in which *all* dealers are pricewatchers, that is the full attention case ($\mu_D = \mu_F = 1$). The analysis is more complex than in the benchmark case as dealers in one security extract information about the factor that is unknown to them from the price of the other security. The information content of prices depends on the mapping between the price of each security and the factors, which is endogenous. Hence, we consider (linear) rational expectations equilibria of the model, i.e., equilibria in which dealers' beliefs regarding the mapping between prices and factors are correct.

Formally, a linear rational expectations equilibrium is a set of prices $\{p_{j1}^*\}_{j\in\{D,F\}}$ such that

$$p_{j1}^* = R_{j1}\delta_j + B_{j1}u_j + A_{j1}\delta_{-j} + C_{j1}u_{-j}, \tag{4.2}$$

and p_{j1}^* clears the market of asset j for each realization of $\{u_j,\ \delta_j,\ u_{-j}, \delta_{-j}\}$ when dealers anticipate that clearing prices satisfy equation (4.2) and choose their trading strategies accordingly. The pressure exerted by a demand shock in market j on the price in this market is measured by

$$\frac{\partial p_j}{\partial u_j} = B_{j1}. \tag{4.3}$$

Thus, B_{j1} measures the "illiquidity of market j" in the full attention case. Index "1" is used to refer to the equilibrium when $\mu_D = \mu_F = 1$.

Proposition 1. *With full attention, there always exists a noisy, linear rational expectations equilibrium. At any equilibrium,* $R_{j1} = 1$, $C_{j1} = A_{j1}B_{-j1}$ *and*

$$\begin{aligned} A_{D1} &= d\gamma_D((d^2+\sigma_\eta^2)B_{F1}^2\sigma_{u_F}^2+\sigma_\eta^2)^{-1}B_{D1} \geq 0 && (4.4)\\ A_{F1} &= \gamma_F(B_{D1}^2\sigma_{u_D}^2)^{-1}B_{F1} \geq 0 && (4.5)\\ B_{j1} &= B_{j0}(1-\rho_{j1}^2), && (4.6) \end{aligned}$$

where $\rho_{D1}^2 \equiv ((\sigma_\eta^2+d^2)(1+B_{F1}^2\sigma_{u_F}^2))^{-1}d^2$ *and* $\rho_{F1}^2 \equiv (1+B_{D1}^2\sigma_{u_D}^2)^{-1}$.

To interpret Proposition 1, let $\omega_j = \delta_j + B_{j1}u_j$. The previous proposition and equation (4.2) imply that, in a linear rational expectations equilibrium, the price of security j can be written as follows

$$p_j = \omega_j + A_{j1}\omega_{-j}, \text{ for } j \in \{F, D\}, \tag{4.7}$$

or

$$p_j = \omega_j + A_{j1}(p_{-j} - A_{j1}\omega_j) = (1 - A_{j1}A_{-j1})\omega_j + A_{j1}p_{-j}, \text{ for } j \in \{F, D\}. \tag{4.8}$$

That is, the price of security j is positively related to the price of the other security since $A_{j1} \geq 0$. Now, consider a dealer in, say security D. From the price of security F, he can infer a noisy signal, $\omega_F \equiv \delta_F + B_F u_F$, about the realization of factor δ_F since $(p_F - A_F p_D)$ is proportional to ω_F (equation (4.8)). Thus, in a linear rational expectations equilibrium, observing $\{\delta_j,\ p_j, p_{-j}\}$ is informationally equivalent to observing $\{\delta_j,\ \omega_{-j}\}$. We deduce that ω_{-j} is a sufficient statistic for the price information available to pricewatchers operating in security j.

The informativeness of the price information available to dealers in security j is measured by ρ_{j1}^2, the squared correlation between the payoff of security j and the price information (ω_{-j}) available to pricewatchers in this security. We obtain

$$\rho_{D1}^2 = \frac{E[v_D\omega_F\,|\delta_D]^2}{\mathrm{Var}[v_D\,|\delta_D]\mathrm{Var}[\omega_F]} = \frac{d^2}{(\sigma_\eta^2+d^2)(1+B_{F1}^2\sigma_{u_F}^2)}, \tag{4.9}$$

$$\rho_{F1}^2 = \frac{E[v_F\omega_D\,|\delta_F]^2}{\mathrm{Var}[v_F\,|\delta_F]\mathrm{Var}[\omega_D]} = \frac{1}{1+B_{D1}^2\sigma_{u_D}^2}. \tag{4.10}$$

For instance, ρ_{D1}^2 determines the informativeness of the price of security F about the payoff of security D for pricewatchers in security D. To see this, remember that the information set of pricewatchers in security j is $\{\delta_j, \omega_{-j}\}$, so that the precision of their forecast about the payoff of security j is $\mathrm{Var}[v_j|\delta_j,\omega_{-j}]^{-1}$. In the absence of price information, the precision of this forecast would be $\mathrm{Var}[v_j|\delta_j]^{-1}$. The additional information conveyed by the price of security $-j$ for dealers in security j is

$$\mathrm{Var}[v_j|\delta_j,\omega_{-j}]^{-1} - \mathrm{Var}[v_j|\delta_j]^{-1} = \mathrm{Var}[v_j|\delta_j]^{-1}\left(1-\rho_{j1}^2\right)^{-1} - \mathrm{Var}[v_j|\delta_j]^{-1} = \frac{\rho_{j1}^2}{1-\rho_{j1}^2},$$

where the second equality follows from the fact that all random variables have a normal distribution. Hence, the higher is ρ_{j1}^2, the more informative is the price of security $-j$ for dealers in security j. For brevity, we often refer to ρ_{j1}^2 as the informativeness of the price of security $-j$.

As expected, when $d = 0$, $\rho^2_{D1} = 0$ since the price of security F does not convey information to dealers in security D (v_D is independent from ω_F).

Using Proposition 1, equation (4.9) and equation (4.10), we obtain that

$$B_{D1} = B_{D0}(1 - \rho^2_{D1}) = f_1(B_{F1}; \gamma_D, \sigma^2_\eta, d, \sigma^2_{u_F}), \tag{4.11}$$

$$B_{F1} = B_{F0}(1 - \rho^2_{F1}) = g_1(B_{D1}; \gamma_F, \sigma^2_{u_D}), \tag{4.12}$$

with

$$f_1(B_{F1}; \gamma_D, \sigma^2_\eta, d, \sigma^2_{u_F}) = \frac{\sigma^2_\eta}{\gamma_D} + \frac{d^2 B^2_{F1}\sigma^2_{u_F}}{\gamma_D(1 + B^2_{F1}\sigma^2_{u_F})},$$

$$g_1(B_{D1}; \gamma_F, \sigma^2_{u_D}) = \frac{B^2_{D1}\sigma^2_{u_D}}{\gamma_F(1 + B^2_{D1}\sigma^2_{u_D})}.$$

Proposition 1 shows that all coefficients in the equilibrium price function can be expressed as functions of B_{D1} and B_{F1}. Thus, the number of linear rational expectations equilibria is equal to the number of pairs $\{B_{D1}, B_{F1}\}$ solving the system of equations (4.11) and (4.12). In general, we cannot find analytical solutions for this system of equations and characterize equilibria in closed-form. However, we can solve for the equilibria numerically. In Figure 2 we illustrate the determination of the equilibrium by plotting the functions $f_1(\cdot)$ and $g_1(\cdot)$ for specific values of the parameters. The equilibria are at the points where the curves representing these functions intersect. In panel (a) we set $\gamma_j = d = 1$, $\sigma_{u_j} = 2$, and $\sigma_\eta = .2$. In this case, we obtain three equilibria: one with a low level of illiquidity, one with a medium level of illiquidity and one with a relatively high level of illiquidity. In panel (b) and (c), we pick values of σ_η or d such that the correlation between the payoffs of securities D and F is smaller ($\sigma_\eta = 1$ in panel (b) while $d = 0.9$ in panel (c)). In this case, we obtain a unique equilibrium

This pattern is more general: when d is low relative to σ^2_η, the model has a unique rational expectations equilibrium whereas otherwise it can have up to three equilibria. The reason for this pattern is as follows. When $d = 0$, dealers in security D do not use the information contained in the price of security F. Hence, the illiquidity of security D only depends on its "fundamentals" (γ_D and σ^2_η). Thus, in a rational expectations equilibrium, the beliefs of dealers in security F regarding the liquidity of security D are uniquely defined. In contrast, when $d > 0$, the illiquidity of security D depends on its fundamentals *and* on the beliefs of its dealers about the illiquidity of security F, which themselves depend on the beliefs of dealers in this security about the illiquidity of security D. This circularity can lead to multiple equilibria.

When d is low relative to σ^2_η, factor δ_F plays a relatively small role in the determination of the payoff of security D. Thus, the beliefs of dealers D about the liquidity of security F play a relatively minor role in the determination of the liquidity of security D. As a consequence the equilibrium is unique, as shown in Corollary 1.

Corollary 1. *With full attention and*

$$\sigma^2_\eta \geq 4d^2, \tag{4.13}$$

there is a unique linear rational expectations equilibrium.

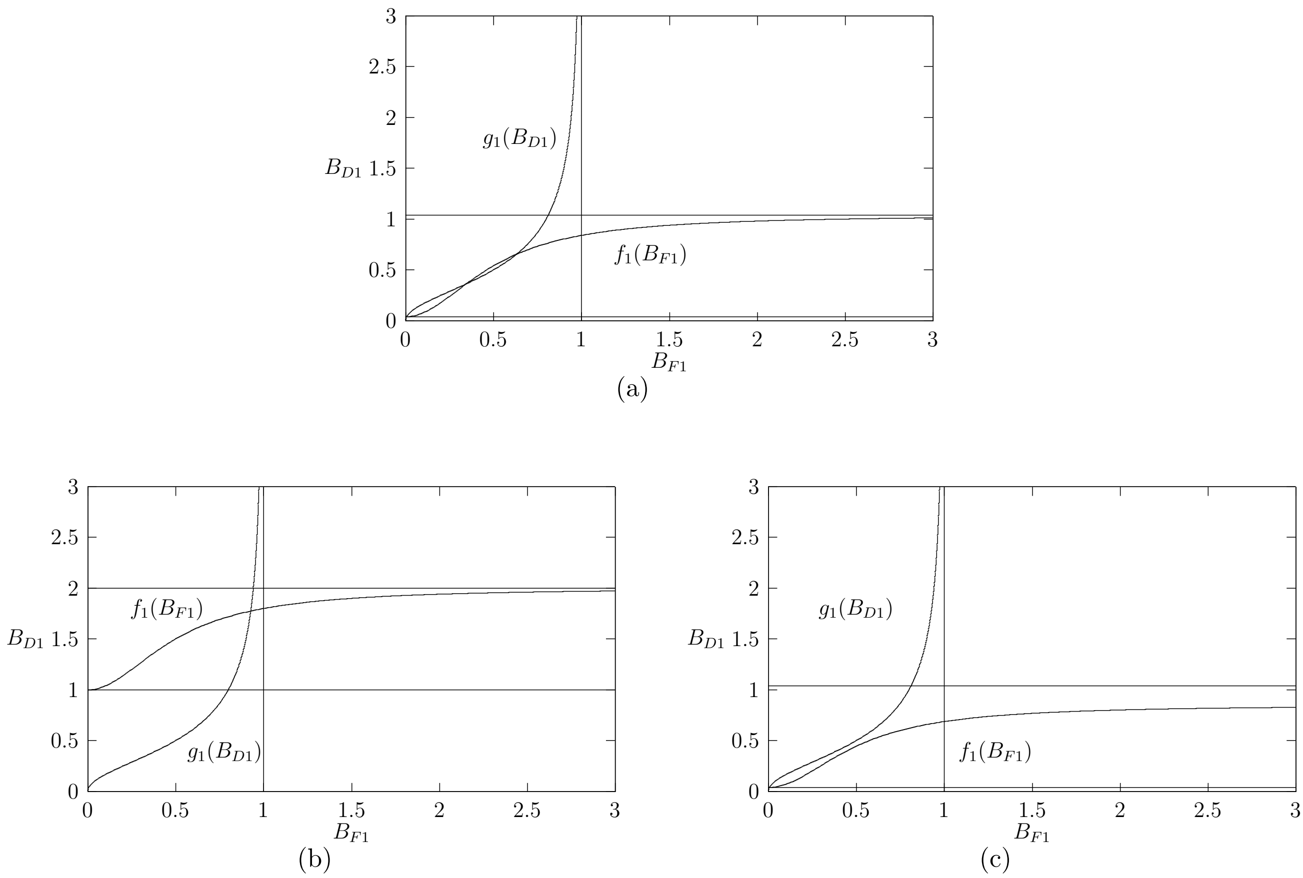


Figure 2: Equilibrium determination with full attention (no adverse selection): multiplicity (panel (a)) and uniqueness (panel (b) and (c)). Parameters' values are as follows: $\gamma_j = d = 1$, $\sigma_{u_j} = 2$, and $\sigma_\eta = .2$ (panel (a)), while in panel (b) we set $\sigma_\eta = 1$ and in panel (c) we set $d = 0.9$.

When the condition in the above result does not hold, d is high relative to σ_η^2, and up to three equilibria can be obtained. For instance, consider the case in which securities F and D have identical payoffs ($d = 1$ and $\sigma_\eta^2 = 0$). Moreover, assume that $\gamma_F = \gamma_D = \gamma$, $\sigma_{u_j}^2 = \sigma_u^2$. We refer tho this case as the *symmetric case* since the markets for the two securities are perfectly identical. It is clearly restrictive but its analysis is useful to gain intuition on the properties of the model. In the symmetric case, if $\sigma_u^2 > 4\gamma^2$, there are three possible equilibria with three different levels of illiquidity: low, medium or high. In each equilibrium the level of illiquidity is identical in the two markets since they are symmetric. We thus drop the market index and denote this level by B^{L*}, B^{M*}, and B^{H*} in the low, medium or high illiquidity equilibrium respectively obtaining

$$B^{H*} = \frac{\sigma_u + (\sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma\sigma_u}, \tag{4.14}$$

$$B^{M*} = \frac{\sigma_u - (\sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma\sigma_u}, \tag{4.15}$$

$$B^{L*} = 0 \tag{4.16}$$

In the low illiquidity equilibrium, prices in each market are fully revealing since $B^{L*} = 0$.[9]

The model can feature multiple equilibria ("liquidity regimes") but all equilibria share common properties. First, as $B_{j1} = B_{j0}(1 - \rho_{j1}^2)$ (Proposition 1) and $\rho_{j1}^2 \leq 1$, we immediately obtain that, in all equilibria, the illiquidity of the market is lower with full attention than with no attention. Moreover, in all equilibria, factors that make the price of security $-j$ more informative make the illiquidity of security j smaller, as shown in the next corollary.

Corollary 2. *The markets for securities D and F are less illiquid with full attention than with no attention, i.e., $B_{j1} \leq B_{j0}$. Moreover, with full attention, an increase in the informativeness of the price of security $-j$ for dealers in security j makes security j more liquid:*

$$\frac{\partial B_{j1}}{\partial \rho_{j1}^2} \leq 0. \tag{4.17}$$

The intuition for this result is simple. By watching the price of another security, dealers learn information. Hence, they face less uncertainty about the payoff of the security in which they are active. For this reason, with full attention, dealers require a smaller premium to absorb a given demand shock and this premium decreases with the informativeness of prices.

Price movements in security j are driven both by news about factor δ_j and demand shocks. The contribution of demand shocks to price variations becomes relatively higher when security j becomes more illiquid. As a consequence the price of security j becomes less informative for dealers in other markets when security j becomes more illiquid, as shown in the next corollary.

Corollary 3. *With full attention, an increase in the illiquidity of security j makes its price less informative for dealers in security $-j$:*

$$\frac{\partial \rho_{-j1}^2}{\partial B_{j1}} \leq 0. \tag{4.18}$$

Corollaries 2 and 3 imply that, in all equilibria and in contrast to the benchmark case, an exogenous change in the illiquidity of one market (due for instance to an increase in dealers' risk tolerance in this market) affects the illiquidity of the other market. We call this effect a *liquidity spillover*.

To see this, consider the effect of an increase in the risk tolerance of dealers in security D.[10] The immediate effect of this increase is to make security D more liquid as in the benchmark case. Hence, its price becomes more informative for dealers in security F (Corollary 3), which then becomes more liquid (Corollary 2) because inventory risk for dealers in security F is smaller

[9] A fully revealing equilibrium obtains for the following reason. Suppose that dealers in security F and D share their information on the factors. As $\sigma_\eta^2 = 0$, they face no uncertainty on the payoff of the security in which they make the market. Thus, the case in which dealers in security j expect the price of the other security to be fully revealing is self-fulfilling. Indeed, in this case, dealers in security j behave as if they were facing no risk and therefore security j is perfectly liquid ($B_{j1} = 0$). This makes the price of security j fully revealing as expected by dealers in security $-j$. The fully revealing equilibrium disappears if $\sigma_\eta^2 > 0$.

[10] More generally, in our model, a variation in risk tolerance of dealers in one security should be more broadly interpreted as variations in the cost of liquidity provision for dealers in one asset class. These variations may be due to risk tolerance, inventory limits or financing constraints for dealers in this asset class. The important point is that they do not directly affect dealers in other asset classes.

when they are all better informed. Thus, the improvement in the liquidity of security D spreads to liquidity F, although security F experiences no change in its liquidity fundamentals.

More formally, consider the system of equations (4.11) and (4.12). Other things equal, an increase in the risk tolerance of dealers in security D makes this security more liquid since $\partial f_1/\partial \gamma_D < 0$. In turn this improvement spreads to security F because $\partial g_1/\partial B_{D1} \neq 0$. More generally, any exogenous change in the illiquidity of security D will spill over to security F because $\partial g_1/\partial B_{D1} \neq 0$. Similarly, liquidity spillovers operate *from security F to security D* if and only if $\partial f_1/\partial B_{F1} \neq 0$. The sign of these liquidity spillovers is determined by the sign of $\partial g_1/\partial B_{D1}$ and $\partial f_1/\partial B_{F1}$.

Corollary 4. *With full attention, liquidity spillovers are always positive, i.e., $\partial f_1/\partial B_{F1} \geq 0$ and $\partial g_1/\partial B_{D1} \geq 0$.*

When $d = 0$, the price of security F conveys no information to dealers in security D ($\rho^2_{D1} = 0$) and there is no spillover from security F to security D ($\partial f_1/\partial B_{F1} = 0$). When $d > 0$, dealers in each market learn information from the price in the other market and, for this reason, liquidity spillovers operate in both directions (from D to F and vice versa).

Now consider again the effect of a small reduction in the risk tolerance of dealers in security D. The direct effects of this reduction on the illiquidity of security D and F are measured by $(\partial f_1/\partial \gamma_D) < 0$ and $(\partial g_1/\partial B_{D1})(\partial f_1/\partial \gamma_D) < 0$, respectively. The total effect however will be larger if learning is two sided. Indeed, in this case, the change in illiquidity for security F feeds back positively on the change in illiquidity for security D which in turn feeds back positively into a further change in illiquidity for security F... Formally this chain of effects entails total differentiation of the system of equations (4.11) and (4.12) and yields:

$$\frac{dB_{D1}}{d\gamma_D} = \frac{\partial f_1}{\partial \gamma_D}\left(1 + \frac{\partial g_1}{\partial B_{D1}}\frac{\partial f_1}{\partial B_{F1}} + \left(\frac{\partial g_1}{\partial B_{D1}}\frac{\partial f_1}{\partial B_{F1}}\right)^2 + \cdots\right) \tag{4.19}$$

$$\frac{dB_{F1}}{d\gamma_D} = \frac{\partial g_1}{\partial B_{D1}}\frac{\partial f_1}{\partial \gamma_D}\left(1 + \frac{\partial g_1}{\partial B_{D1}}\frac{\partial f_1}{\partial B_{F1}} + \left(\frac{\partial g_1}{\partial B_{D1}}\frac{\partial f_1}{\partial B_{F1}}\right)^2 + \cdots\right), \tag{4.20}$$

and therefore potentially amplifies the direct effect. Let

$$\kappa \equiv \frac{1}{(1 - (\partial g_1/\partial B_{D1})(\partial f_1/\partial B_{F1}))}, \tag{4.21}$$

and assume that $(\partial g_1/\partial B_{D1})(\partial f_1/\partial B_{F1}) \leq 1$ so that $\kappa \geq 1$. Now, consider a reduction in the risk tolerance of dealers in security D. Computing the sum of the terms in the parentheses in (4.19) and (4.20) yields

$$\underbrace{\frac{dB_{D1}}{d\gamma_D}}_{\text{Total Effect}} = \kappa \underbrace{\frac{\partial f_1}{\partial \gamma_D}}_{\text{Direct Effect}} < 0,$$

$$\underbrace{\frac{dB_{F1}}{d\gamma_D}}_{\text{Total Effect}} = \kappa \underbrace{\frac{\partial g_1}{\partial B_{D1}}\frac{\partial f_1}{\partial \gamma_D}}_{\text{Direct Effect}} < 0.$$

Security D becomes less liquid when the risk tolerance of dealers in this security becomes lower ($\partial f_1/\partial\gamma_D < 0$). As a consequence, its price becomes less informative. Hence, dealers in security F face more uncertainty and security F becomes less liquid, although its liquidity fundamental (γ_F) is *unchanged*. When learning is two sided ($d > 0$), the reduction in liquidity for security F that follows the initial reduction in liquidity for security D feeds back positively on the liquidity of security D. This triggers a vicious circle by which the initial effects of the reduction in the risk tolerance of dealers in security D are amplified by a factor $\kappa > 1$. If κ is very large, this amplification effect can lead to much larger changes in the liquidity of both markets than what would be observed in the absence of cross-asset learning (see Figure 3, panel (a)). When learning is one sided, there is no amplification effect since the reduction in liquidity for security F does not feedback on the liquidity of security D ($\kappa = 1$ if $d = 0$; see Figure 3, panel (b)).

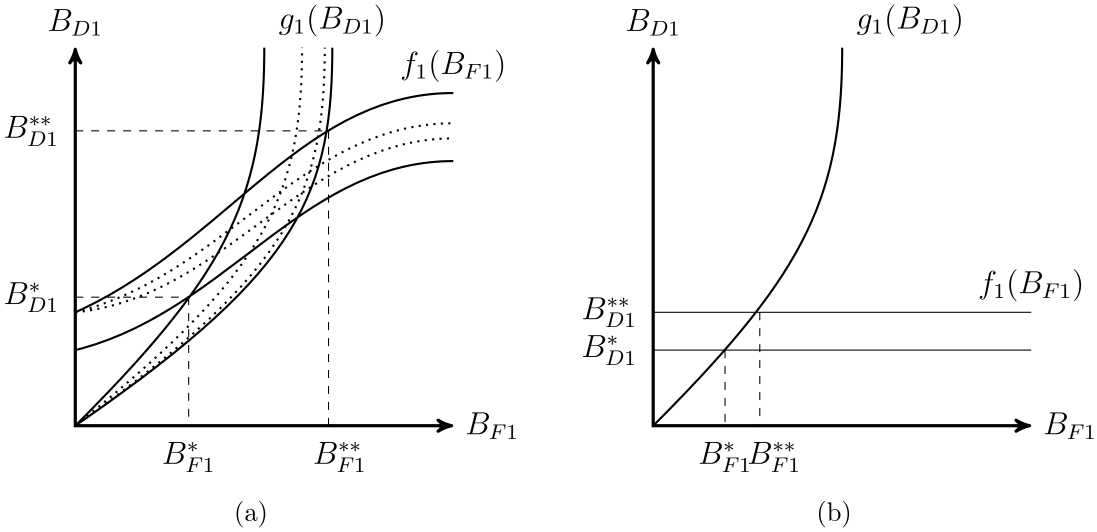


Figure 3: Full attention and positive liquidity spillovers. Starting from an equilibrium in which the illiquidity of the two markets is given by B_{D1}^* and B_{F1}^*, a reduction in risk tolerance for dealers in security D causes the function $f_1(B_{F1})$ to shift upwards. The adjustment towards the new equilibrium (denoted by the point $(B_{D1}^{**}, B_{F1}^{**})$) differs depending on whether learning is two-sided (panel (a)) or one-sided (panel (b)).

This discussion is valid when $\kappa \geq 1$. The next corollary shows that there always exist an equilibrium in which this condition holds.

Corollary 5. *With full attention, there always exist an equilibrium in which $\kappa > 1$. In any equilibrium with $\kappa > 1$:*

1. *An increase in the risk aversion of dealers in security j leads to a drop in the liquidity of both securities.*

2. *An increase in the idiosyncratic risk of security D leads to a drop in the liquidity of both securities.*

If the equilibrium is unique, then $\kappa > 1$.

With multiple equilibria, there may be cases in which $\kappa < 1$. In this case, the total effect of a change in the liquidity fundamental of one security can be counter-intuitive. For instance, a reduction in risk tolerance for dealers in security D can lead to an reduction in the illiquidity of both securities. An example of this fact is represented by the equilibrium with intermediate illiquidity in the symmetric case in which $\kappa < 1$. In this equilibrium, a reduction in dealers' risk tolerance ultimately leads to a reduction in illiquidity.

4.3 Limited attention, adverse selection, and negative co-movements in liquidity

We now turn to the more general case in which $0 < \mu_D \leq 1$ and $0 < \mu_F \leq 1$. That is we allow for limited attention by dealers in either security. In this case, the spillover mechanism that we described in the previous section still operates. However, the direction of these spillovers is ambiguous. Indeed, an improvement in liquidity in one security, say j, has a priori an ambiguous effect on the liquidity of the other security. On the one hand, as explained previously, it makes the price of security j more informative and thereby it has a positive effect on the liquidity of security $-j$. On the other hand, it also increases the informational advantage of pricewatchers in security $-j$ relative to other dealers in this market. This effect is a source of adverse selection among dealers in security $-j$ and has therefore a negative impact on the liquidity of this security. Hence, liquidity spillovers can a priori be positive or negative. We now identify conditions on the parameters so that these spillovers are positive.

To this end, we first need to solve for the linear rational expectations equilibrium when only a fraction of dealers in either security are pricewatchers. A linear rational expectations equilibrium is a set of prices $\{p_j^*\}_{j\in\{D,F\}}$ such that

$$p_j^* = R_j\delta_j + B_j u_j + A_j\delta_{-j} + C_j u_{-j}, \tag{4.22}$$

and p_j^* clears the market of asset j for each realizations of $\{u_j, \delta_j, u_{-j}, \delta_{-j}\}$ when dealers anticipate that clearing prices satisfy equation (4.22) and choose their trading strategies accordingly.

Proposition 2. *With limited attention (i.e., $0 < \mu_D \leq 1$ and $0 < \mu_F \leq 1$), there always exists a noisy, linear rational expectations equilibrium. At any equilibrium $R_j = 1$, $C_j = A_j B_{-j}$,*

$$A_D = d\gamma_D\mu_D((d^2+\sigma_\eta^2)B_F^2\sigma_{u_F}^2 + \sigma_\eta^2)^{-1}B_D \geq 0 \tag{4.23}$$

$$A_F = \gamma_F\mu_F(B_D^2\sigma_{u_D}^2)^{-1}B_F \geq 0, \tag{4.24}$$

and

$$B_j = B_{j0}(1-\rho_j^2) \times \frac{\gamma_j^2\mu_j\rho_j^2 + \sigma_{u_j}^2\mathrm{Var}[v_j|\delta_j](1-\rho_j^2)}{\gamma_j^2\mu_j^2\rho_j^2 + \sigma_{u_j}^2\mathrm{Var}[v_j|\delta_j](1-\rho_j^2)(1-\rho_j^2(1-\mu_j))}, \tag{4.25}$$

where $\rho_D^2 \equiv ((\sigma_\eta^2+d^2)(1+B_F^2\sigma_{u_F}^2))^{-1}d^2$ and $\rho_F^2 \equiv (1+B_D^2\sigma_{u_D}^2)^{-1}$.

The interpretation of Proposition 2 is identical to the interpretation we offered for Proposition 1. In particular, parameter ρ_j^2 determines the informativeness of the signal, $\omega_{-j} = \delta_{-j} + B_{-j}u_{-j}$, that pricewatchers in security j obtain from the price of security $-j$.

As pricewatchers' trading strategy depends on the information they obtain from watching the price of security $-j$ (i.e., ω_{-j}), the equilibrium price in security j partially reveals pricewatchers' private information.[11] In fact, equation (4.22) shows that observing the price of security j is informationally equivalent to observing $\hat{\omega}_j \equiv A_j\omega_{-j} + B_j u_j$. Thus, in equilibrium, the information set of outsiders, $\{\delta_j,\, p_j\}$, is informationally equivalent to $\{\delta_j,\, \hat{\omega}_j\}$. In what follows, we sometimes refer to ω_{-j} as pricewatchers' price signal and $\hat{\omega}_j$ as outsiders' price signal. Clearly, outsiders' price signal is less precise than pricewatchers' price signal, which means that outsiders in security j are at an informational disadvantage compared to pricewatchers. Intuitively, as they are less informed, outsiders will hold a smaller position than pricewatchers when the asset value is high and a higher position than pricewatchers when the asset value is low. This bias in their portfolio holdings is a source of adverse selection, which is absent when all dealers are pricewatchers. This new effect is key to understand why liquidity spillovers may be negative in the limited attention case.

Substituting ρ_D^2 and ρ_F^2 by their expressions in equation (4.25), we express B_j as a function of B_{-j}. Formally, we obtain:

$$B_D = f(B_F; \mu_D, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) \tag{4.26}$$

$$B_F = g(B_D; \mu_F, \gamma_F, \sigma_{u_D}^2), \tag{4.27}$$

where functions $f(\cdot)$ and $g(\cdot)$ are given in the appendix for brevity.[12]

The linear rational expectations equilibria are completely characterized by the solution(s) of this system of equations and when $\mu_D = \mu_F = 1$, these solutions are those obtained in the full attention case analyzed in Section 4.2. As explained previously, there might be multiple equilibria and we cannot in general characterize these equilibria in closed-form. However, when $d = 0$, the analysis is simplified since liquidity spillovers operate only from security D to F (formally, $\rho_D^2 = 0$). Hence, in this case, the level of illiquidity in security D is as in the benchmark case ($B_D = \sigma_\eta^2/\gamma_D$) and the level of illiquidity in security F is readily obtained by substituting this expression for B_D in equation (4.25). Proceeding in this way we obtain the following result.

Corollary 6. *With limited attention and one-sided learning ($d = 0$), there is a unique linear rational expectations equilibrium where the levels of illiquidity of securities D and F are*

$$B_D = \sigma_\eta^2/\gamma_D \tag{4.28}$$

$$B_F = \frac{B_D^2\sigma_{u_D}^2(B_D^2\sigma_{u_F}^2\sigma_{u_D}^2 + \mu_F\gamma_F^2)}{\gamma_F(\mu_F^2\gamma_F^2(1 + B_D^2\sigma_{u_D}^2) + B_D^2\sigma_{u_D}^2\sigma_{u_F}^2(\mu_F + B_D^2\sigma_{u_D}^2))}. \tag{4.29}$$

In this equilibrium:

---

[11] Pricewatchers' demand can be written as

$$x_j^I(p_j, \omega_{-j}) = a_j^I\,(E[v_j \mid \delta_j, p_{-j}] - p_j) = a_j^I(\delta_j - p_j) + b_j^I\omega_{-j},$$

where expressions for coefficients a_j^I and b_j^I are provided in the proof of Proposition 2.

[12] See equations (A.20) and (A.21) in the Appendix.

1. *If $\sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F] \geq \gamma_F^2$, liquidity spillovers are always positive. If $\sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F] < \gamma_F^2$, positive liquidity spillovers arise if and only if $\mu_F > \hat{\mu}_F$.*

2. *If $\sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F] \geq \gamma_F^2$, an increase in attention by dealers in F improves the liquidity of market F. If $\sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F] < \gamma_F^2$, a liquidity improvement occurs if and only if $\mu_F > \hat{\hat{\mu}}_F$,*

where explicit values for $\hat{\mu}_F$ and $\hat{\hat{\mu}}_F$ are provided in the appendix.

When $\mu_D = \mu_F = 1$, the corollary describes the equilibrium obtained with full attention and one sided learning. Differently from the case with full attention, when $\mu_j < 1$, liquidity spillovers can be negative. To see why, consider an increase in the risk aversion of the dealers

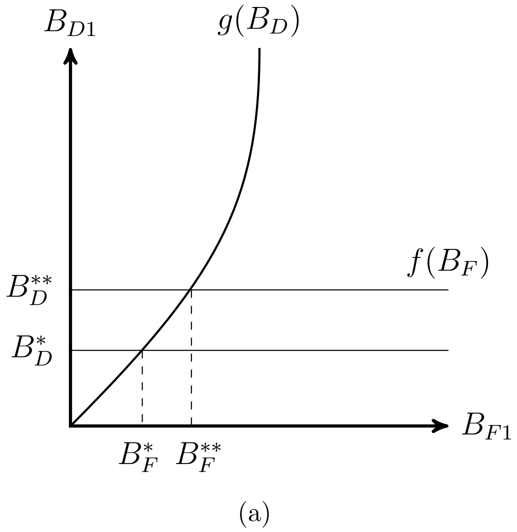


Figure 4: With limited attention, liquidity spillovers can be positive (panel (a)) or negative (panel (b)). Starting from an equilibrium in which the illiquidity of the two markets is given by B_D^* and B_F^*, a decrease in risk tolerance for dealers in security D causes the function $f(B_F)$ to shift upwards. The adjustment towards the new equilibrium (denoted by the point (B_D^{**}, B_F^{**})) differs depending on whether the adverse selection effect is weaker or stronger than the uncertainty effect (respectively, panel (a) and panel (b)). In the former case, the illiquidity of both assets increases. In the latter case, the function $g(B_D)$ is non-monotone in B_D. Hence, while the illiquidity of asset D increases, asset F experiences an illiquidity *reduction*.

operating in security D. Owing to (4.28), this increase makes security D less liquid and therefore less informative for pricewatchers in security F ($\partial\rho_F^2/\partial B_D < 0$). Thus, uncertainty about the payoff of security F increases. As when $\mu_D = \mu_F = 1$, this "uncertainty effect" increases the illiquidity of security F. But, now, there is a countervailing effect. Indeed, as pricewatchers' private information is less precise, their informational advantage is smaller. As a consequence, outsiders' exposure to adverse selection is smaller as well, which tends to increase the liquidity of security F. According to the above result, the latter effect can prevail if dealers in F speculate very aggressively on their signal, demand shocks are small and the payoff features little residual uncertainty (i.e., $\sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F] < \gamma_F^2$). In this case, provided the fraction of pricewatchers μ_F is small, adverse selection is strong, and a liquidity reduction in market D *improves* the liquidity of market F (see Figure 4, panel (b)). Conversely, when $\sigma^2_{u_F}\text{Var}[v_F|\delta_F] \geq \gamma_F^2$, adverse selection is weak, and positive liquidity spillovers always obtain (see Figure 4, panel (a)). A similar intuition explains the effect of an increase in μ_F. Summarizing, the liquidity spillover from security D to security F is positive only if the uncertainty effect dominates the adverse selection effect. The next corollary provides a sufficient condition on the parameters of the model for this to be the case for all values of d.

Corollary 7. *Let*

$$\overline{\mu}_j = \max\left\{0, 1 - \frac{\sigma^2_{u_j}\text{Var}[v_j|\delta_j]}{\gamma_j^2}\right\}, \text{ for } j \in \{D, F\}. \tag{4.30}$$

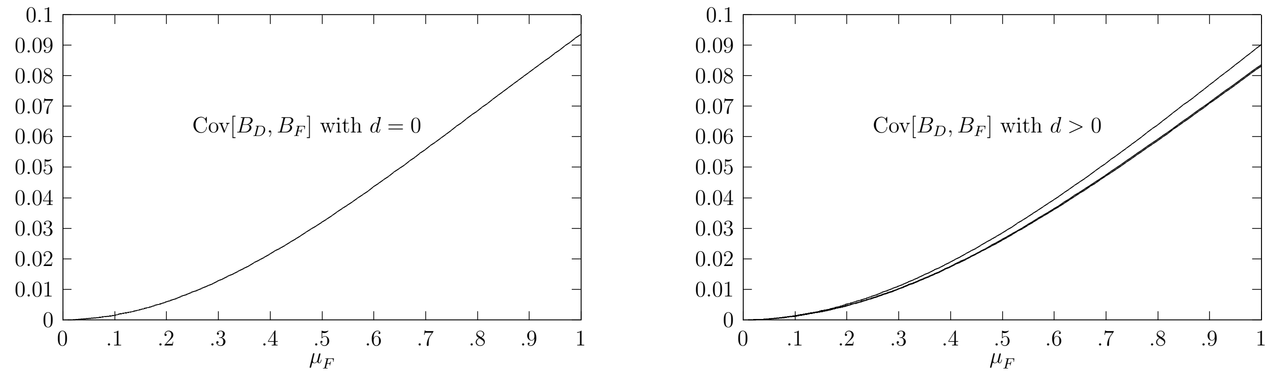
If $\mu_D \in [\overline{\mu}_D, 1]$ and $\mu_F \in [\overline{\mu}_F, 1]$ then liquidity spillovers from security D to security F and vice versa are positive.

Thus, the model will feature positive liquidity spillovers (and therefore positive co-movements in liquidity) if the level of attention in each market is higher than a threshold, $\overline{\mu} = \max\{\overline{\mu}_D, \overline{\mu}_F\}$. This threshold is equal to zero if the aggregate risk exposure of dealers in both markets ($\sigma^2_{u_j}\text{Var}[v_j|\delta_j]$) is high enough relative to their risk tolerance. Graphically, positive liquidity spillovers entail an adjustment process in the functions that determine the equilibrium levels for B_F and B_D similar to the one depicted in Figure 3.

Intuitively, positive liquidity spillovers generate positive co-movements in illiquidity across-securities. Also, the strength of these co-movements should be greater when dealers' attention to prices of other securities is high since spillovers exist if and only if a fraction of dealers in each market watch the prices of other securities. To illustrate these points we use the following numerical example.

Example 1. For a given value of μ_F, we compute the illiquidity of securities F and D assuming that γ_D is uniformly distributed in $[0.5, 1]$ and setting $\sigma^2_{u_F} = \sigma^2_{u_D} = 1/4$, $\sigma^2_\eta = 4$, $\gamma_F = 1/2$ (note that for these parameter values $\overline{\mu}_j = 0$). We then compute the covariance between the resulting equilibrium values for B_D and B_F. Figure 5 (panel (a)) reports the value of this covariance when $\mu_F \in \{0, 0.01, 0.02, \ldots, 1\}$ for $\mu_D \in \{0.1, 0.9\}$. In panel (b) we repeat the same exercise with $d = 0.9$. First, observe that in both cases, the covariance between the illiquidity of securities D and F is positive if and only if $\mu_F > 0$. That is, co-movements in liquidity require the presence of pricewatchers in our model. Moreover, as expected this covariance is stronger when the fraction of pricewatchers is higher. This is seen in two ways. On the one hand, the covariance between the illiquidity of securities F and D increases in μ_F. On the other hand, when $d > 0$ (panel (b)), the covariance between the illiquidity of the two securities is higher when $\mu_D = 0.9$ (light curve) than when $\mu_F = 0.1$ (bold curve), for all values of $\mu_F > 0$.

The fraction of pricewatchers in a given security is itself a determinant of the liquidity of this security. Consider the effect of a small increase in the fraction of insiders in market j, holding *fixed the level of the illiquidity in the other market.* This effect is measured by $(\partial f/\partial \mu_D)$ or



(a) (b)

Figure 5: Attention levels as an engine of comovement in illiquidity. The figure displays the covariance between the illiquidities in the F and D markets as a function of μ_F when $d = 0$ (panel (a)) and $d = 0.9$ (panel (b)). In panel (b) the covariance between the illiquidity of the two securities is higher when $\mu_D = 0.9$ (light curve) than when $\mu_F = 0.1$ (bold curve), for all values of $\mu_F > 0$.Other parameter values are $\sigma_{u_F} = \sigma_{u_D} = 1/2$, $\sigma_\eta = 2$, $\gamma_F = 1/2$, and $\mu_D \in \{0.1, 0.9\}$.

$(\partial g/\partial \mu_F)$. We call this effect the *direct* effect of a change in the fraction of pricewatchers in market j. The sign of this direct effect is determined by two opposite forces. On the one hand, an increase in the fraction of pricewatchers in market j raises the exposure to adverse selection for dealers who remain uninformed about the price of security $-j$. On the other hand, more dealers bear relatively low inventory carrying cost because they are less uncertain about the payoff of security j. The first effect raises illiquidity while the second effect decreases illiquidity. As shown in Corollary 8, the second effect always prevails when $\sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j] > \gamma_j^2$.

Corollary 8. *With limited attention, if*

$$\sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j] > \gamma_j^2,$$

for $j \in \{D, F\}$ then, other things equal, an increase in attention by dealers in security j reduces the illiquidity of this security ($(\partial f/\partial \mu_D) < 0$ and $(\partial g/\partial \mu_F) < 0$). Furthermore, there is always an equilibrium in which an increase in attention by dealers in security j reduces the illiquidity of both securities in equilibrium.

Example 2. We illustrate these findings with numerical simulations. We set $\sigma_\eta = 0.77$, $\sigma_{u_j} = 1$ and $d = \gamma_j = 1$. In this case we check that a unique equilibrium always arises for the chosen parameter values. In Figure 6, we plot the illiquidity of security D as a function of $\mu_D \in \{0.001, 0.002, \ldots, 1\}$ when $\mu_F = 0.5$ (panel (a)) and $\mu_F = 0.9$ (panel (b)) when B_F is fixed at its equilibrium value for $\mu_D = 0.001$ (bold curve) and when B_F adjusts to its equilibrium value for each value of μ_D (dotted curve). Thus, the bold curve represents the direct effect of a change in the fraction of pricewatchers in security D (i.e., the effect holding constant the liquidity of security F) while the dotted curve represents the evolution of the equilibrium value of the illiquidity of security D, after accounting for spillover effects. The difference between the two curves shows the amount by which spillover effects magnify the direct effect of a change in attention on illiquidity.

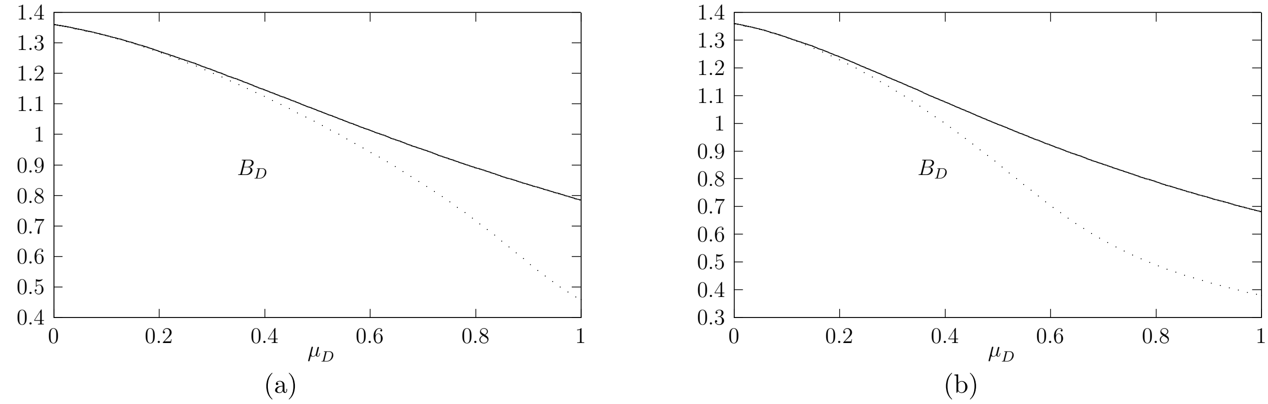


Figure 6: The figure displays the illiquidity of security D as a function of μ_D when $\mu_F = 0.5$ (in panel (a)) and when $\mu_F = 0.9$ (panel (b)) when B_F is fixed at its equilibrium value for $\mu_D = 0.001$ (bold curve) and when instead it adjusts to its equilibrium value for each value of μ_D (dotted curve). The difference between the two curves shows the amount by which spillover effects magnify the direct effect of a change in attention on illiquidity. Parameters' values are as follows: $\sigma_{u_j} = 1$, $\sigma_\eta = 0.77$ and $d = \gamma_j = 1$.

As argued above with limited attention liquidity spillovers can also be negative. In Corollary 6 we provide exact conditions for this to happen when learning is one-sided ($d = 0$). With two-sided learning we cannot obtain an analytical characterisation, but numerical examples can be built to show that an intuition similar to the one developed in the case $d = 0$ goes through in this case as well. Similarly to what done in Figure 4, in Figure 7 we graphically analyse the impact of a reduction in the risk tolerance of dealers in market D when outsiders in market F suffer from a strong adverse selection effect, so that a negative spillover runs from security D to security F. Since with $d > 0$, liquidity spillovers are two-sided, $f(B_F)$ is now positively sloped, in contrast to the case $d = 0$ (compare panel (a) and (b) of Figure 7). Figure 8 illustrates the result of a numerical simulation in which an increased participation of pricewatchers in market D triggers a negative liquidity spillover from market D to market F. Parameters' values are as follows: $\sigma_{u_F} = .1$, $\sigma_{u_D} = 1$, $\gamma_F = 1$, $\gamma_D = .1$, $d = 1$, $\mu_F = .4$, and $\sigma_\eta = .5$. In this case, there are few pricewatchers in market F who trade aggressively and the demand of liquidity traders is relatively small. As a result, adverse selection risk is high in this market, which leads to a negative liquidity spillover. That is, as μ_D gets larger, B_D falls but B_F increases.

Table 2 provides a summary of our main results so far.

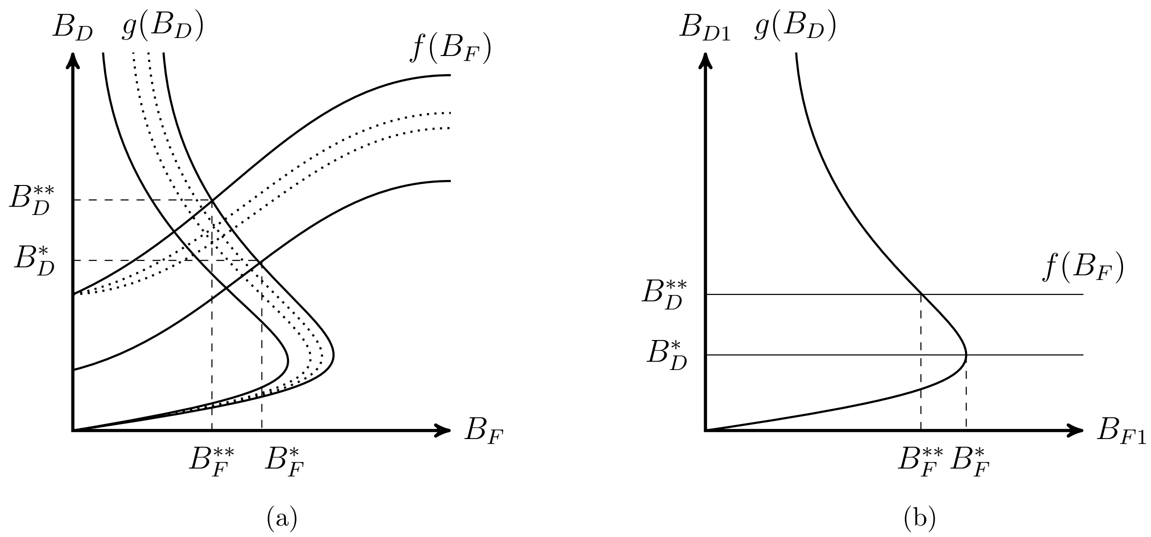


Figure 7: Negative liquidity spillovers with two- and one-sided learning (respectively, panel (a) and panel (b)). Starting from an equilibrium in which the illiquidity of the two markets is given by B_D^* and B_F^*, a decrease in risk tolerance for dealers in security D causes the function $f(B_F)$ to shift upwards. Note that the adjustment towards the new equilibrium differs depending on whether there is two- or one-sided learning.

Attention		Sign of liquidity spillovers	$\uparrow \mu_j$ on B_j	$\uparrow \mu_j$ on B_{-j}
No		No liquidity spillovers	No effect	No effect
Limited	$\sigma_{u_j}^2 \text{Var}[v_j \mid \delta_j] > \gamma_j^2$	Positive liquidity spillovers	Negative	Negative[a]
	$\sigma_{u_j}^2 \text{Var}[v_j \mid \delta_j] \leq \gamma_j^2$	Ambiguous[b]	Ambiguous[b]	Ambiguous[b]
Full		Positive liquidity spillovers	N.A.	N.A.

Table 2: A summary of results.

[a]Provided $\kappa > 1$.
[b]Positive for μ_D and μ_F large enough.

5 Endogenous attention

We now endogenize the level of attention that so far we have taken as being exogenous. To this end we introduce a cost of attention and we study the fraction of dealers in each security who decide to pay attention to the other security. We first consider the case in which liquidity spillovers operate only from security D to security F (i.e., $d = 0$). In this case, as expected, the level of attention by dealers in security F is inversely related to the cost of attention. Hence, variations in attention (and co-movements in liquidity between securities D and F) are ultimately driven by variations in the cost of attention. Then, in Section 5.2, we consider the case in which liquidity spillovers operate in both directions. In this case, surprisingly, the economics of the attention decision is significantly different. Indeed, the value of attention for dealers in one security can increase both in the level of attention by dealers in the *same* security and dealers in the other security. As a consequence, dealers' attention decisions reinforce each

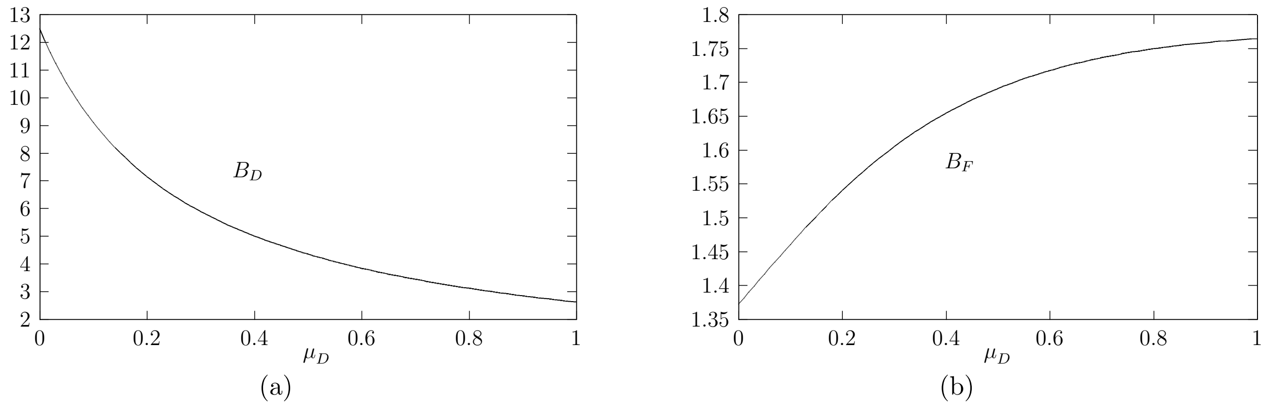


Figure 8: Negative liquidity spillovers. Parameters' values are as follows: $\sigma_{u_F} = .1$, $\sigma_{u_D} = 1$, $\gamma_F = 1$, $\gamma_D = .1$, $d = 1$, $\mu_F = .4$, and $\sigma_\eta = .5$.

other *and* multiple equilibria with differing levels of attention can arise for the same level of the cost of attention.

5.1 Attention decisions with one-sided learning

Let $\phi_j(\mu_j, B_{-j})$ be the value of the information contained in the price of security $-j$ for dealers in security j when a fraction μ_j of dealers in security j are informed about the price of security $-j$. This value is the maximum fee that a dealer in security j is willing to pay to get informed on p_{-j}. Thus, it solves:

$$E\left[U\left((v_j - p_j)\, x_j^I - \phi_j\right)\right] = E\left[U\left((v_j - p_j)\, x_j^O\right)\right]. \tag{5.1}$$

In general, the solution to this equation depends on the level of illiquidity in security $-j$ since this level determines the informational content of the price of security $-j$. We stress this feature by explicitly writing ϕ_j as a function of the illiquidity of security $-j$: $\phi_j(\mu_j, B_{-j})$.

When $d = 0$, there is a unique rational expectations equilibrium for all values of μ_F (see Corollary 6) and dealers in security D learn no information from the price of security F. Thus, in this case, $\phi_D(\mu_D, B_F) = 0$ and no dealers in security D monitor the price of security F since this is worthless, i.e., $\mu_D = 0$. This feature considerably simplifies the analysis. In particular it implies that the level of illiquidity in security D is fixed at $B_D = \sigma_\eta^2/\gamma_D$ for all possible values of μ_F. Hence, to simplify notation, in this section we write $\phi_F(\mu_F, B_D)$ simply as $\phi_F(\mu_F)$.

Using the specification of dealers' utility functions and the fact that all variables have a normal distribution, we obtain that[13]

$$\phi_F(\mu_F) = \frac{\gamma_F}{2} \ln\left(\frac{\mathrm{Var}[v_F|\delta_F, \hat{\omega}_F]}{\mathrm{Var}[v_F|\delta_F, \omega_D]}\right) > 0. \tag{5.2}$$

In equilibrium, when $\mu_F > 0$, all dealers in security F obtain information about factor δ_D. Pricewatchers extract this information from the price of security D and obtain a signal ω_D

[13]Our expression for the value of information is standard in models of information acquisition with normally distributed variables and CARA utility functions (see for instance Admati and Pfleiderer (1986)). Thus, for brevity we omit the derivation of this result, which can be obtained upon request.

about factor δ_F. The price information privately observed by pricewatchers leaks partially through the price of security F as pricewatchers trade on this information, which conveys a signal $\hat{\omega}_F$ to outsiders. This signal is less informative than the signal obtained by pricewatchers since price movements in security F are also affected by the aggregate liquidity shock in this security. For this reason, pricewatchers can form a more precise forecast of the payoff of security F than outsiders, that is $\mathrm{Var}[v_F|\delta_F, \hat{\omega}_F] > \mathrm{Var}[v_F|\delta_F, \omega_D]$ and the value of being a pricewatcher is always strictly positive. Intuitively, the value of monitoring the price of security D for dealers in security F decreases in the fraction of pricewatchers in security F because the leakage effect is stronger when the fraction of pricewatchers in security F is higher. We establish this result in the next corollary.

Proposition 3. *If $d = 0$,*

$$\phi_F(\mu_F) = \frac{\gamma_F}{2} \ln\left(1 + \frac{\sigma_{u_F}^2 \sigma_{u_D}^2 B_D^2}{\gamma_F^2 \mu_F^2 (1 + B_D^2 \sigma_{u_D}^2) + \sigma_{u_F}^2 \sigma_{u_D}^4 B_D^4}\right). \tag{5.3}$$

with $B_D = \sigma_\eta^2/\gamma_D$. Thus, the value of monitoring the price of security D for dealers in security F decreases in the fraction of pricewatchers in security F.

Hence, with one sided learning, the value of acquiring price information declines with the fraction of dealers buying this information, as usual in models of information acquisition (e.g., Grossman and Stiglitz (1980) or Admati and Pfleiderer (1986)). Let C be the cost for dealers in one security to monitor the price of the other security. We refer to this cost as the *cost of attention*. In centralized markets with real time dissemination of price information, this cost includes the fee charged by data vendors and the time cost of monitoring this information. It can also be interpreted as the cost of receiving market data in advance of other dealers. For instance, in reality, some participants pay a "co-location" fee to trading platforms. In exchange, they obtain the right to position their computers close to platforms' servers and, in this way, they possess a split second advantage in accessing trade and quote data. Let $\mu_F^*(C)$ be the fraction of dealers in security F who decide to pay this cost. As $\phi_F(\mu_F)$ decreases in μ_F, there are three possible cases:

1. If $\phi_F(1) > C$, then the value of monitoring the price of security D for dealers in security F exceeds the cost of monitoring even when all dealers pay the cost of monitoring. Thus, $\mu_F^*(C) = 1$.
2. If $\phi_F(0) < C$, then the value of monitoring the price of security D for dealers in security F is always lower than the cost of monitoring. Thus, $\mu_F^*(C) = 0$.
3. Otherwise, the equilibrium fraction of pricewatchers is such that dealers in security F are just indifferent between monitoring the price of security D or not. That is, $\mu_F^*(C)$ is the unique solution of $\phi_F(\mu_F) = C$.

We obtain the following result.

Proposition 4. *With one sided learning ($d = 0$), the fraction $\mu_F^*(C)$ of dealers in security F who monitor the price of security D in equilibrium decreases in the cost of attention. This fraction is:*

1. $\mu_F^*(C) = 0$, *if* $C > \overline{C}$.
2. $\mu_F^*(C) = \sqrt{\frac{\sigma_{u_F}^2 \sigma_{u_D}^2 B_D^2 (1 - B_D^2 \sigma_{u_D}^2 (e^{2C/\gamma_F} - 1))}{\gamma_F^2 (1 + B_D^2 \sigma_{u_D}^2)(e^{2C/\gamma_F} - 1)}}$, *if* $\underline{C} \leq C \leq \overline{C}$.
3. $\mu_F^*(C) = 1$, *if* $C < \underline{C}$,

where closed-form solutions for the thresholds $\underline{C}$ and $\overline{C}$ are given in the proof of the proposition and $B_D = \sigma_\eta^2/\gamma_D$.

As explained in Section 4.3, the fraction of pricewatchers in a security is one determinant of the illiquidity of this security. Thus, ultimately, the cost of attention is one illiquidity fundamental of this security since it determines the fraction of dealers who chooses to be pricewatchers in this security. The next corollary describes the effect of a change in the cost of attention on the illiquidity of security F.

Corollary 9. *With one sided learning ($d = 0$):*

1. *If $\sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F] \geq \gamma_F^2$ then the illiquidity of security F increases in the cost of attention for dealers active in this security.*
2. *If $\sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F] < \gamma_F^2$, there exists a value of $C^* \in (\underline{C}, \overline{C})$ such that the illiquidity of security F increases in the cost of attention for dealers active in this security when $C \leq C^*$ and decreases in the cost of attention otherwise (the closed-form solution for C^* is given in the proof of the corollary).*

A decrease in the cost of attention leads to an increase in the fraction of pricewatchers. As explained in Section 4.3, this evolution has an ambiguous effect on the illiquidity of security F. On the one hand, more attention reduces the uncertainty on the payoff of security F. On the other hand, inattentive dealers are more exposed to adverse selection if the attention of their competitors increases. As shown in Corollary 6, the first effect always dominates when $\sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F] \geq \gamma_F^2$. Thus, in this case, a reduction in the cost of monitoring for dealers in security F always improves the liquidity of this security. When $\sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F] < \gamma_F^2$, the second effect dominates as long as the fraction of inattentive dealers remains high, i.e., when C is greater than C^*. Indeed in this case, only a few dealers are well informed and, as a result the leakage effect is small. Hence, inattentive dealers are more exposed to adverse selection and market liquidity deteriorates. Figure 9 illustrates the impact that a change in the cost of attention has on the fraction of pricewatchers, illiquidity, and the value of information with one-sided learning.

5.2 Attention decisions with two sided learning

We now consider the case in which $d > 0$, so that dealers in each security can learn information from the price of the other security. In this case, our main finding is that the value of price monitoring by dealers in a given market can be increasing in the fraction of pricewatchers in this market. This finding is counter-intuitive since usually the value of financial information declines with the fraction of investors acquiring this information (see Grossman and Stiglitz (1980) or Admati and Pfleiderer (1986)). The value of price information has this property when liquidity spillovers operate only one way (D to F), as we have just shown in Proposition 4. Otherwise price information is *special*: its value can increase in the number of investors who use it. As we shall see the main reason for this counter-intuitive result is that the value of price information tends to be higher for securities that are more liquid and securities tend to be more liquid when the fraction of pricewatchers is large.

Proceeding as in the previous section, we deduce that the value of monitoring the price of security $-j$ for dealers in security j is

$$\begin{aligned}\phi_j(\mu_j, B_{-j}(\mu_j, \mu_{-j})) &= \frac{\gamma_j}{2}\ln\frac{\mathrm{Var}[v_j|\delta_j,\hat{\omega}_j]}{\mathrm{Var}[v_j|\delta_j,\omega_{-j}]}\\ &= \frac{\gamma_j}{2}\ln\left(1+\frac{\sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j](1-\rho_j^2)\rho_j^2}{\mu_j^2\gamma_j^2\rho_j^2+\sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j](1-\rho_j^2)^2}\right),\end{aligned} \tag{5.4}$$

where we stress the fact that the illiquidity of each market in equilibrium is a function of pricewatchers in either market.[14] For a fixed fraction of pricewatchers in market $-j$, we have

$$\frac{d\phi_j}{d\mu_j} = \underbrace{L_j}_{\text{Leakage effect}} + \underbrace{\Lambda_j}_{\text{Feedback effect}}. \tag{5.5}$$

with $L_j \equiv (\partial\phi_j/\partial\mu_j)$ and $\Lambda_j \equiv (\partial\phi_j/\partial B_{-j})(\partial B_{-j}/\partial\mu_j)$. Thus, the total effect of an increase in the fraction of pricewatchers in security j on the value of being a pricewatcher is the sum of two effects: the leakage effect (that we described in the previous section) and the feedback effect, which is new. To understand this effect, consider an increase in the fraction of pricewatchers in security D (the reasoning is symmetric for an increase in μ_F). When $d > 0$, this increase affects the liquidity of security D and thereby the liquidity of security F since spillovers operate both ways (see Corollary 8). In turn, the change in the liquidity of security F feeds back on the value of monitoring this security since, as explained before, it affects the informativeness of the price of security F for dealers in security D. The change in the value of information due to this feedback effect is measured by Λ_D.[15]

The total effect of an increase in the fraction of pricewatchers in security j on the value of information in this market is positive if and only if the feedback effect outweighs the leakage

[14] To obtain (5.4) recall that $a_j^I = \gamma_j/\mathrm{Var}[v_j|\delta_j,\omega_{-j}]$ and $a_j^O = \gamma_j/\mathrm{Var}[v_j|\delta_j,\hat{\omega}_j]$ and use the expressions for a_j^I and a_j^O obtained in the appendix. For example, using (A.3), and (A.18) yields ϕ_D.

[15] When $d = 0$, the feedback effect does not operate for either security since the illiquidity of security D is fixed at $B_D = B_{D0} = (\sigma^2_\eta/\gamma_D)$ for all values of the other parameters. Thus, $(\partial B_D/\partial\mu_F) = 0$ and $(\partial B_F/\partial\mu_D) = 0$ which implies that $\Lambda_D = \Lambda_F = 0$.

effect:

$$\Lambda_j > -L_j > 0. \tag{5.6}$$

Obviously, this condition requires the feedback effect to be positive, which is a possibility when $\sigma_{u_j}^2 \text{Var}[v_j|\delta_j] \geq \gamma_j^2$. To see this, consider again the value of monitoring security F for dealers in security D. When $\sigma_{u_D}^2 \text{Var}[v_D|\delta_D] \geq \gamma_D^2$, as shown in Corollary 8, an increase in the fraction of pricewatchers in security D reduces the illiquidity of security F $((\partial B_F/\partial \mu_D) < 0)$. As a consequence, the price of security F becomes more informative for dealers in security D and the value of monitoring this price is higher, at least for some parameter values $((\partial \phi_D/\partial B_F) < 0)$. Thus, the feedback effect for security D is positive: $\Lambda_D > 0$.

In traditional models of financial information acquisition, investors' decisions to buy information are "strategic substitutes": information acquisition by a larger number of investors reduce the value of being informed. This effect is captured here by the leakage effect. When positive, the feedback effect works in the opposite direction. If this effect more than compensates the leakage effect, dealers' decisions to buy price information are "strategic complements": more pricewatchers make the value of being a pricewatcher higher, not smaller!

We have not been able to delineate the exact set of parameters under which the feedback effect dominates the leakage effect (i.e., condition (5.6) holds true). However, numerical simulations show that this set of parameters is not empty. To see this, consider Figure 10. Panel (a) on this figure plots the value of monitoring security F for pricewatchers in security D (i.e., $\phi_D(\mu_D, B_F)$) for two values of μ_F ($\mu_F = 0.1$ and $\mu_F = 0.9\}$). Other parameter values are $\gamma_F = \gamma_D = 1$, $d = 1$, $\sigma_\eta = 1$ and $\sigma_{u_F} = \sigma_{u_D} = 1$. In both cases the value of monitoring security F increases with the fraction of pricewatchers in security D, which means that the feedback effect dominates the leakage effect.

Now consider the effect of a change in the fraction of pricewatchers located in market $-j$ on the value of monitoring this market for dealers in asset j. This effect is measured by

$$\frac{d\phi_j}{d\mu_{-j}} = \left(\frac{\partial \phi_j}{\partial B_{-j}} \frac{\partial B_{-j}}{\partial \mu_{-j}}\right). \tag{5.7}$$

As shown in Corollary 8, an increase in the fraction of pricewatchers in, say, security D reduces the illiquidity of this security $((\partial B_D/\partial \mu_D) < 0)$ if $\sigma_{u_D}^2 \text{Var}[v_D|\delta_D] \geq \gamma_D^2$. In turn this effect makes the price of security D more informative for dealers in security F and increases the value of monitoring this price for dealers in security F. In this case, $(d\phi_F/d\mu_D) > 0$. That is, an increase in the fraction of pricewatchers in security D makes the value of monitoring the price of security D higher for dealers in security F. In other words, the decisions of dealers located in different markets to follow each other markets are strategic complements.

Figure 10 illustrates this effect. First, consider panel (a) again. It shows that the value of monitoring security F for dealers in security D is higher, other things equal when $\mu_F = 0.9$ than when $\mu_F = 0.1$, that is $(d\phi_D/d\mu_F) > 0$. Moreover, panel (b) shows that an increase in the fraction of pricewatchers in security D makes the value of monitoring security D higher for dealers in security F.

Thus, price information is special because dealers' decisions to buy this information can reinforce each other both in the *same* market and across *different* markets. The model shows that this happens in two distinct ways: (i) the value of being informed about the price of another security can increase in the fraction of dealers who follow this security ("same market complementarity") and (ii) the value of being informed about the price of another security can increase in the fraction of pricewatchers in this security ("cross market complementarity"). Both sources of complementarity in dealers' moniting decisions are absent when $d = 0$ and they do not necessarily both operate when $d > 0$ (in particular the leakage effect may prevail over the feedback effect even though the cross-market complementarity operates).

Now consider whether a dealer in market j should become a pricewatcher. In making this decision, the dealer takes the fraction of pricewatchers in both markets as given. If $\phi_j(\mu_j, B_{-j}(\mu_D, \mu_F)) > C$, it is optimal for the dealer to be a pricewatcher since the value of monitoring the price in the other market is higher than the cost. If $\phi_j(\mu_j, B_{-j}(\mu_D, \mu_F)) < C$, it is optimal for the dealer not to monitor the price in the other market and finally for $\phi_j(\mu_j, B_{-j}(\mu_D, \mu_F)) = C$, the dealer is just indifferent. Given these observations, the equilibrium fractions of pricewatchers in each market, (μ_D^*, μ_F^*), are displayed in Table 3.

μ_j^*, μ_{-j}^*	When
$\mu_j^* = \mu_{-j}^* = 1$	$\phi_j(1, B_{-j}(1,1)) > C$ for $j \in \{D, F\}$
$\mu_j^* = 1$, $\mu_{-j}^* \in (0,1)$	$\phi_j(1, B_{-j}(1, \mu_{-j}^*)) > C$ and $\phi_{-j}(\mu_{-j}^*, B_{-j}(1, \mu_{-j}^*)) = C$
$\mu_j^*, \mu_{-j}^* \in (0,1)$	$\phi_j(\mu_j^*, B_{-j}(\mu_j^*, \mu_{-j}^*)) = C$ for $j \in \{D, F\}$
$\mu_j^* = 0, \mu_{-j}^* \in (0,1)$	$\phi_j(0, B_{-j}(0, \mu_{-j}^*)) < C$ and $\phi_{-j}(\mu_{-j}^*, B_{-j}(1, \mu_{-j}^*)) = C$
$\mu_j^*, \mu_{-j}^* = 0$	$\phi_j(0, B_{-j}(0,0)) < C$ for $j \in \{D, F\}$.

Table 3: The equilibrium fraction of pricewatchers in markets j and $-j$.

Intuitively, complementarities in attention decisions among dealers located in different markets lead to multiple equilibria for the levels of attention. Indeed, these complementarities imply that the value of cross-market monitoring will be relatively high when the fraction of pricewatchers in both markets is high and relatively low when the fraction of pricewatchers in both markets is low. Thus, for intermediate values of the cost of monitoring, there is room for multiple equilibria with various levels of market integration for the same values of the parameters (in particular the correlation of the payoffs of the two securities being fixed).

As an example, consider the parameter values of Figure 5 again and suppose $C = 0.06$. In this case, there are three possible pairs of equilibrium values for the levels of attention in each market: (i) $\mu_D^* = \mu_F^* = 1$, (ii) $\mu_D^* = 0$, $\mu_F^* = 1$ and (iii) $\mu_D^* \simeq 0.3$, $\mu_F^* = 1$. In all these equilibria, all dealers in security F pay attention to the price of security D. In contrast, *for the same parameter values*, we can have an equilibrium in which dealers in security D do not follow security F ($\mu_D^* = 0$), an equilibrium in which all dealers in security D follow security

F ($\mu_D^* = 1$) or an equilibrium in which only a fraction of dealers in security D buy price information on security F ($\mu_D^* \simeq 0.3$). Thus, for the same fundamentals, dealers in security D can appear to neglect the information contained in the price of security F or to be relatively very sensitive to this information.

We may also have situations in which, for the *same parameter values*, the markets for the two securities appear fully segmented because dealers in either market pay no attention to the other market ($\mu_D^* = \mu_F^* = 0$) or fully integrated because all dealers are pricewatchers ($\mu_D^* = \mu_F^* = 1$). To see this, consider the case in which the two markets are perfectly symmetric: $\gamma_F = \gamma_D = \gamma$, $d = 1$, $\sigma_\eta = 0$ and $\sigma_{u_F} = \sigma_{u_D} = \sigma_u$. Remember that in this case, there are two non-fully revealing rational expectations equilibria if $\mu_D = \mu_F = 1$. For the discussion, we focus on the high illiquidity equilibrium in which the level of illiquidity in markets D and F is B^{H*} (given in equation (4.14)). Using Proposition 2 and equation (5.4), we obtain that:

$$\phi_j(\mu_j, B_{-j}) = \frac{\gamma}{2} \ln \left(1 + \frac{B_{-j}^2 \sigma_u^4}{\gamma^2 \mu_j^2 (1 + B_{-j}^2 \sigma_u^2) + B_{-j}^4 \sigma_u^6}\right). \tag{5.8}$$

Using this expression, we obtain the following result.

Proposition 5. *Suppose that $\gamma_F = \gamma_D = \gamma$, $d = 1$, $\sigma_\eta = 0$ and $\sigma_{u_F} = \sigma_{u_D} = \sigma_u$. The value of monitoring prices in market $-j$ for dealers in market j is strictly higher when $\mu_D = \mu_F = 1$ than when $\mu_D = \mu_F = 0$, that is, $\phi_j(1, B^{H*}) > \phi_j(0, B_{j0})$ for $j \in \{H, L\}$.*

Thus, the value of price monitoring is higher when all dealers are pricewatchers than when no dealer is a pricewatcher. By symmetry, we have $\phi_F(1, B^{H*}) = \phi_D(1, B^{H*})$ and $\phi_F(0, B_{F0}) = \phi_D(0, B_{D0})$. That is, the value of price information is identical in each market in the full attention case and in the no attention case, respectively. Let ϕ_0 be the value of price information in the no attention case and let ϕ_1 be the value of price information in the full attention case. We obtain the following result.

Proposition 6. *If $\phi_0 < C < \phi_1$, $\mu_D^* = \mu_F^* = 1$ and $\mu_D^* = \mu_F^* = 0$ are two possible equilibrium levels of attention when dealers' monitoring decisions are endogenous.*

Proof. Suppose that $\mu_D^* = \mu_F^* = 1$. Then in this case, the value of monitoring market j for a dealer in security $-j$, given the actions of other dealers, is ϕ_1. As this value is higher than C, monitoring is optimal. Hence $\mu_D^* = \mu_F^* = 1$ is an equilibrium. Now suppose that $\mu_D^* = \mu_F^* = 0$. Then in this case, the value of monitoring market j for a market-maker in market $-j$, given the actions of other dealers, is ϕ_0. As this value is lower than C, not monitoring is optimal. Hence $\mu_D^* = \mu_F^* = 0$ is an equilibrium. □

Thus, for the *same* parameters value, the markets for securities F and D can be either fully integrated (all dealers in each market account for the price information available in the other market) or fully segmented. As an illustration, suppose that $\sigma_\delta = \sigma_u = 1$, $\gamma = 1/2$. Figure 11 plots the value of price monitoring when the fraction of pricewatchers in each market is μ. In this case, we have

$$\phi_0 = \frac{\gamma}{2} \ln \left(1 + \frac{\gamma^2}{\sigma_u^2}\right) \approx 0.055, \;\; \phi_1 = \frac{\gamma}{2} \ln \left(1 + \frac{(B^{H*})^2 \sigma_u^4}{\gamma^2 (1 + (B^{H*})^2 \sigma_u^2) + (B^{H*})^4 \sigma_u^6}\right) \approx 0.127.$$

Thus, for any value of $C \in [0.055, 0.127]$, the markets for securities F and D can be either fully segmented or fully integrated, depending on whether dealers in both markets coordinate on the high or the low attention equilibrium. The liquidity of both markets and the informativeness of prices are higher if dealers coordinate on the high attention equilibrium. The figure also shows that the value of price monitoring is always increasing in the fraction of pricewatchers. Thus, for any $C \in (0.055, 0.127)$, there is a third symmetric equilibrium with an intermediate level of attention in which $\mu_j^* = \phi_j^{-1}(C)$.[16] But this equilibrium is unstable. Interestingly, in this case, the markets can remain segmented even if the cost of attention decreases, unless it falls below $C = 0.055$.

In summary, when learning is two sided, the value of price information can increase in the fraction of pricewatchers. This property means that dealers' decisions to monitor the price of another security are complements both within and across markets. That is, they reinforce each other. As a consequence, multiple equilibria with differing levels of attention are sustainable and two securities may appear segmented even though the correlation of their payoffs is high and the cost of monitoring is relatively low. This result suggests that coordination failures in monitoring decisions can per se be a source of market segmentation.

6 Implications

Our model describes a new mechanism to explain liquidity spillovers, i.e., co-variation in liquidity of different securities. This mechanism works as follows. A liquidity reduction in, say security D, makes its price less informative for dealers active in security F. This effect increases the uncertainty borne by pricewatchers in security F but reduces the exposure to adverse selection risk of outsiders in security F. Thus, the deterioration in the liquidity of security D affects the liquidity of security F but the direction of the effect is unambiguously positive only when the fraction of pricewatchers in security F is high enough. Otherwise the liquidity of security F may improve. In all cases, spillovers arise through dealers' attention to prices of other securities.

How to test empirically whether this mechanism explains, at least partially, commonalities in liquidity? One possible empirical strategy consists in considering exogenous changes in the cost of attention due, either to structural changes in market design or to technological changes. We illustrate this idea with two thought experiments.

6.1 Information driven liquidity spillovers and TRACE

The first experiment is as follows. Initially, the markets for securities D and F are opaque so that the cost of obtaining information on the prices of securities D and F is high. In this case, $\mu_D^{before} = 0$ and $\mu_F^{before} = 0$. Now suppose that the market for security D becomes transparent

[16]In this example, as we focus on the case in which the two markets are symmetric, we only consider the case in which the fractions of pricewatchers in each market are equal. There might be other non symmetric equilibria.

while the market for security F remains opaque so that $\mu_D^{after} = \mu_D^{before} = 0$ but $\mu_F^{after} > 0$ since transparency reduces the cost of acquiring information on the price of security D for dealers in security F. The model makes several predictions about the effects of this change in market design.

Namely, if $\sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F] \geq \gamma_F^2$, the liquidity of security F should increase after the market for security D becomes transparent (see Corollary 8), even though the market structure for security F is identical before and after the change affecting the other security. Moreover, co-variation in liquidity between securities D and F should be positive and greater than before the change in market design as explained in Section 4.3 (see Figure 5).

If instead, $\sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F] < \gamma_F^2$ and the fraction of pricewatchers located in security F remains small after the reform then the liquidity of security F can decrease. The reason is that the transparency of security D reduces uncertainty for pricewatchers in security F but it exposes other dealers in this security (the outsiders) to adverse selection. Moreover, in this case, liquidity spillovers from security D to security F may be be negative.

The implementation of the TRACE system in the U.S. corporate bond market is a field experiment close to the thought experiment we just described. Until 2002, the U.S. corporate bond market was very opaque: the price of each transaction was known only to the parties involved in the transaction. This situation changed when the SEC required dissemination of transaction prices (with a delay) for a subset of bonds through a reporting system called TRACE. This requirement initially applied to 498 bonds and was implemented in July 2002. Bessembinder et al. (2006) study the effects of this reform of the bond market on the liquidity of TRACE eligible bonds (security D in our thought experiment) and non-TRACE-eligible bonds (security F).[17] Interestingly, Bessembinder et al. (2006) find an increase in liquidity for non-TRACE eligible bonds with mid- and high-volume but *a decrease* in liquidity for low volume non-TRACE-eligible bonds.

This pattern fits well with our predictions since trading volume in the bond market is a proxy for the size of liquidity trades ($\sigma_{u_F}^2$) in our model. Now, the model predicts that (i) even bonds not affected by the reform should experience a change in their liquidity and (ii) this change is unambiguously positive only if $\sigma_{u_F}^2$ is large enough, as observed by Bessembinder et al. (2006). The model makes the additional prediction, yet untested to our knowledge, that the co-movement in liquidity of high and medium volume non-TRACE-eligible bonds should increase after the implementation of TRACE. In contrast, the co-movement in liquidity for low volume non-TRACE-eligible bonds may have decreased.[18]

[17] Edward et al. (2005) and Goldstein et al. (2007) also consider the effects of greater transparency in the U.S. bond markets. However, they do not analyze the effects of greater transparency on non-eligible bonds.

[18] Bessembinder et al.(2010) also finds that the liquidity of the TRACE eligible bonds increase. This also is consistent with the model. To see this suppose now that both the markets for securities D and F become transparent. If $\sigma_{u_j}^2 \mathrm{Var}[v_j|\delta_j] \geq \gamma_j^2$ for both securities then the liquidity of both securities is higher in the transparent system, for all values of the fraction of pricewatchers (see Table 2).

6.2 Information driven liquidity spillovers and high frequency market making

The recent years have witnessed a growth of so called "high frequency market-makers" (e.g., GETCO, Optiver, etc...). This breed of dealers uses highly automated strategies to post quotes in a variety of securities. Their trading strategy relies in part on using price information available about one security to take positions in other securities. For instance, they may extract information on the systematic risk component of securities prices from index futures and use this information to price other securities.

The case in which $d = 0$ can be used to analyze the effect of high-frequency market-makers. Indeed, in this case we can interpret security D as providing information on a market wide risk factor (δ_D) and security F as a security that loads on this factor and another factor (δ_F). We interpret pricewatchers in security F as high frequency market-makers: they watch in real-time the price of security D and use this information to determine their position in security F.

Now suppose that the cost of monitoring (C) decreases. For instance, high frequency market-markers have access to more efficient information processing technologies or the price of real-time data decreases, either because the cost of co-location declines or because data vendors lower their data fees. In all these cases, the return on price monitoring is higher and therefore the number of high frequency market-makers should increase, as predicted by Proposition 4. If $\sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F] \geq \gamma^2_F$, this entry should improve the liquidity of security F and increase the co-movement in liquidity between security D and F (see Corollary 9 and Figure 5).

However, if $\sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F] < \gamma^2_F$, the scenario is more complex. If $C > C^*$, entry of new high frequency market-makers increases exposure to adverse selection for other dealers in security F and the liquidity of this market drops (see Corollary 9). Moreover, liquidity spillovers can switch sign, which implies that co-movements in liquidity between security F and D can become negative.[19] Indeed, an improvement in liquidity for security D allows pricewatchers in security F to obtain more precise information. Thus, if the fraction of pricewatchers remains small, the risk of adverse selection for outsiders increases and the liquidity of market F drops following an increase in liquidity for security D.

Jovanovic and Menkveld (2010) study entry of a high frequency market-maker in Dutch stocks traded on Chi-X (a European trading platform). They show empirically that following this entry, quotes in Chi-X become relatively more informative on price innovations in the Dutch index futures.[20] Moreover, the liquidity of the stocks in which the high frequency market-maker is active improves. This is consistent with the model when $\sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F] \geq \gamma^2_F$. In this case the model makes the additional prediction that co-movements in liquidity should be higher after entry of the high-frequency market-maker.

[19]Dealers in security D have no information to learn from security F. Thus, the market for security D will *lead* the market for security F. Thus, one way to test our predictions about covariations in liquidity in this case is simply to examine lead-lag relationships between the returns of security F and D.

[20]Hendershott and Riordan (2010) also show empirically that high frequency traders make the market more informationally efficient.

7 Conclusions

In this paper we describe a new mechanism that explains the transmission of liquidity shocks from one security to another ("liquidity spillovers"). We consider a model in which two securities are traded by two different pools of risk averse dealers. As the payoffs of these securities are correlated, dealers in one security can learn information from the price of the other security. As securities' prices are noisier when markets are less liquid, a decline in liquidity in one market spreads to the other market. Liquidity spillovers due to price information transmission appear to be partly responsible for the events of May 6, 2010 (the day of the so called "Flash-Crash"). Indeed, the evocative events narrative drafted by the joint CFTC-SEC Commission on Emerging Regulatory Issues suggests that fundamental traders (our "speculators") failed to countervail the drop in prices during the flash crash because they could not tell immediately which type of events (news or liquidity) did trigger the drop.[21] This is consistent with a basic premise of our analysis.

In addition to describing a new mechanism for liquidity spillovers, the model delivers three novel results:

1. In the presence of two sided learning, cross-asset price monitoring can lead to multiple equilibria with differing levels of liquidity.

2. A decrease in the cost of attention (i.e., an increase in the fraction of dealers monitoring the price of other securities) can increase market illiquidity if it triggers a too small increase in the fraction of dealers who pay attention to the price of other securities.

3. The value of acquiring information on the price of other securities can increase, for some parameter values, with the fraction of dealers buying this information. This possibility implies that dealers' decisions to acquire price information are complements (both within and across markets). For this reason, for the same parameter values, multiple levels of segmentation (high, medium or low) between securities can be sustained in equilibrium depending on whether dealers coordinate on equilibria with high, medium or low attention levels to prices. Thus, the level of segmentation between two securities is in part driven by coordination problems among dealers operating in different securities.

In the last part of the paper, we argue that the model can shed light on recent changes in the transparency of the U.S. corporate bond market and the emergence of computerized trading (so called algorithmic trading). Indeed, these changes can be interpreted as changing the cost of attention. Thus, we can make predictions about the effects of these changes on market liquidity and liquidity commonalities by studying the effect of a change in the cost of attention and the fraction of dealers paying attention to the price of other securities in our model.

[21] "...a number of participants reported that because prices simultaneously fell across many types of securities, they feared the occurrence of a cataclysmic event of which they were not yet aware, and that their strategies were not designed to handle." Findings regarding the market events of May 6, 2010, *Report of the Staffs of the CFTC and SEC to the Joint Advisory Committee on Emerging Regulatory Issues* available at `http://www.sec.gov/news/studies/2010/marketevents-report.pdf`.

In our analysis we take the cost of attention as being exogenous. In reality, part of this cost is determined by pricing decisions of data vendors (Bloomberg, Reuters, exchanges, etc. . .). An interesting extension of our paper would be to endogenize this cost by studying the optimal pricing policy of sellers of price information in our set-up.[22]

[22]Cespa and Foucault (2009) study the optimal pricing policy for a monopolist seller of price information. But they restrict their attention to the case with a single security.

A Appendix

Proof of Proposition 1

The proposition is a special case of Proposition 2, which considers the more general case in which μ_j is not necessarily equal to one. $\square$

Proof of Proposition 2

We show that when the coefficients satisfy the conditions given in Proposition 2, the mapping given in equation (4.25) is a rational expectations equilibrium. Observe that in this case, the price in market j can be written $p_j = \omega_j + A_j\omega_{-j}$, where $\omega_j = \delta_j + B_j u_j$. Thus, $\{\delta_j, \omega_{-j}\}$ is a sufficient statistic for $\{\delta_j, p_{-j}, p_j\}$. Moreover, $\{\delta_j, \hat{\omega}_j\}$ is a sufficient statistic for $\{\delta_j, p_j\}$, where $\hat{\omega}_j = B_j u_j + A_j\omega_{-j}$.

Step 1. Equilibrium in market j.

Insiders' demand function. An insider's demand function in market j, $x_j^I(\delta_j, p_j, p_{-j})$, maximizes

$$E\left[-\exp\left\{-\left((v_j - p_j)x_j^I\right)/\gamma_j\right\}|\delta_j, p_j, p_{-j}\right].$$

We deduce that

$$\begin{aligned} x_j^I(\delta_j, p_j, p_{-j}) &= \gamma\frac{E[v_j|\delta_j, p_{-j}, p_j] - p_j}{\mathrm{Var}[v_j|\delta_j, p_{-j}]} \\ &= a_j^I(E[v_j|\delta_j, p_{-j}, p_j] - p_j), \end{aligned} \tag{A.1}$$

with $a_j^I = \gamma_j \mathrm{Var}[v_j|\delta_j, p_{-j}]^{-1}$.

As $\{\delta_D, \omega_F\}$ is a sufficient statistic for $\{\delta_D, p_F, p_D\}$, we deduce (using well-known properties of normal random variables) that

$$\begin{aligned} E[v_D|\delta_D, p_F, p_D] &= E[v_D|\delta_D, \omega_F] \\ &= \delta_D + \frac{d}{(1 + B_F^2\sigma_{u_F}^2)}\omega_F, \end{aligned} \tag{A.2}$$

and

$$\begin{aligned} a_D^I &= \frac{\gamma_D}{\mathrm{Var}[v_D \mid \delta_D, \omega_F]} \\ &= \gamma_D\left(\frac{1 + B_F^2\sigma_{u_F}^2}{d^2B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2)}\right) \\ &= \frac{\gamma_D}{\mathrm{Var}[v_D|\delta_D](1 - \rho_D^2)}, \end{aligned} \tag{A.3}$$

where $\rho_D^2 \equiv ((\sigma_\eta^2 + d^2)(1 + B_F^2\sigma_{u_F}^2))^{-1}$. Thus,

$$x_D^I(\delta_D, \omega_F) = a_D^I(\delta_D - p_D) + b_D^I\omega_F,$$

where

$$\begin{aligned} b_D^I &= \frac{\gamma_D}{\mathrm{Var}[v_D|\delta_D, \omega_F]}\frac{\mathrm{Cov}[v_D, \omega_F]}{\mathrm{Var}[\omega_F]} \\ &= da_D^I\left(\frac{1}{1 + B_F^2\sigma_{u_F}^2}\right). \end{aligned} \tag{A.4}$$

Similarly, for insiders in market F we obtain

$$x_F^I(\delta_F, \omega_D) = a_F^I(\delta_F - p_F) + b_F^I \omega_D, \tag{A.5}$$

where $\omega_D = \delta_D + B_D u_D$, and

$$a_F^I = \gamma_F \left(\frac{1 + B_D^2 \sigma_{u_D}^2}{B_D^2 \sigma_{u_D}^2} \right), \quad b_F^I = a_F^I \frac{1}{1 + B_D^2 \sigma_{u_D}^2}, \tag{A.6}$$

implying that $B_D > 0$ for an equilibrium to be well defined. Note that similarly to what we have obtained for a_D^I, we can rearrange a_F^I to get

$$a_F^I = \frac{\gamma_F}{\mathrm{Var}[v_F | \delta_F](1 - \rho_F^2)},$$

where $\rho_F^2 \equiv (1 + B_D^2 \sigma_{u_D}^2)^{-1}$.

Outsiders. An outsider's demand function in market j, $x_j^O(\delta_j, p_j)$, maximizes:

$$E\left[-\exp\left\{ -\left((v_j - p_j)\, x_j^O \right) / \gamma_j \right\} | \delta_j, p_j \right].$$

We deduce that

$$\begin{aligned} x_j^O(\delta_j, p_j) &= \gamma_j \frac{E[v_j | \delta_j, p_j] - p_j}{\mathrm{Var}[v_j | \delta_j, p_{-j}]} \\ &= a_j^O (E[v_j | \delta_j, p_j] - p_j), \end{aligned} \tag{A.7}$$

with $a_j^O = \gamma_j \mathrm{Var}[v_j | \delta_j, p_{-j}]^{-1}$.

As $\{\delta_D, \hat{\omega}_D\}$ is a sufficient statistic for $\{\delta_D, p_D\}$, we deduce (using well-known properties of normal random variables) that

$$\begin{aligned} E[v_D | \delta_D, p_D] &= E[v_D | \delta_D, \hat{\omega}_D] \\ &= \delta_D + \frac{d A_D}{A_D^2 (1 + B_F^2 \sigma_{u_F}^2) + B_D^2 \sigma_{u_D}^2} \hat{\omega}_D, \end{aligned} \tag{A.8}$$

and

$$\begin{aligned} a_D^O =& \frac{\gamma_D}{\mathrm{Var}[v_D \mid \delta_D, \hat{\omega}_D]} \\ =& \gamma_D \frac{A_D^2 (1 + B_F^2 \sigma_{u_F}^2) + B_D^2 \sigma_{u_D}^2}{d^2 (A_D^2 B_F^2 \sigma_{u_F}^2 + B_D^2 \sigma_{u_D}^2) + \sigma_\eta^2 (A_D^2 (1 + B_F^2 \sigma_{u_F}^2) + B_D^2 \sigma_{u_D}^2)}. \end{aligned} \tag{A.9}$$

Thus,

$$x_D^O(\delta_D, \hat{\omega}_D) = a_D^O(\delta_D - p_D) + b_D^O \hat{\omega}_D,$$

where

$$\begin{aligned} b_D^O &= \frac{\gamma_D}{\mathrm{Var}[v_D | \delta_D, \hat{\omega}_D]} \frac{\mathrm{Cov}[v_D, \hat{\omega}_D]}{\mathrm{Var}[\hat{\omega}_D]} \\ &= a_D^O \frac{d A_D}{A_D^2 (1 + B_F^2 \sigma_{u_F}^2) + B_D^2 \sigma_{u_D}^2}. \end{aligned} \tag{A.10}$$

Similarly, for market F we obtain:

$$x_F^O(\delta_F, \hat{\omega}_F) = a_F^O(\delta_F - p_F) + b_F^O\hat{\omega}_F, \tag{A.11}$$

where

$$a_F^O = \gamma_F \frac{A_F^2(1 + B_D^2\sigma_{u_D}^2) + B_F^2\sigma_{u_F}^2}{A_F^2 B_D^2\sigma_{u_D}^2 + B_F^2\sigma_{u_F}^2}, \quad b_F^O = a_F^O \frac{A_F}{A_F^2(1 + B_D^2\sigma_{u_D}^2) + B_F^2\sigma_{u_F}^2}. \tag{A.12}$$

Clearing price in market j. The clearing condition in market $j \in \{D, F\}$ imposes

$$\int_0^{\mu_j} x_j^I(\delta_j, p_j, p_{-j})di + \int_{\mu_j}^1 x_j^O(\delta_j, p_j)di + u_j = 0$$

Let $a_j = \mu_j a_j^I + (1 - \mu_j)a_j^O$. Using equations (A.1) and (A.7), we solve for the equilibrium price and we obtain

$$p_j = \delta_j + \left(\frac{\mu b_j^I + (1 - \mu_j)b_j^O A_j}{a_j}\right)\omega_j + \left(\frac{(1 - \mu_j)b_j^O B_j + 1}{a_j}\right)u_j, \tag{A.13}$$

Hence in equilibrium we have: Identifying the parameters in the price, in equilibrium, we must have:

$$B_j = \left(\frac{(1 - \mu_j)b_j^O B_j + 1}{a_j}\right), \ A_j = \left(\frac{\mu b_j^I + (1 - \mu_j)b_j^O A_j}{a_j}\right),$$

which implies:

$$B_j = \frac{1}{a_j - (1 - \mu_j)b_j^O} \tag{A.14}$$

$$A_j = \mu_j B_j b_j^I. \tag{A.15}$$

Hence, at a linear rational expectations equilibrium equations (A.3), (A.4), (A.6), (A.9), (A.10), (A.12), together with (A.14) and (A.15) must be satisfied with $B_j > 0$ (as otherwise the demand function of insiders in market F is not well defined). As these equations must hold for both markets, we have a system of 12 equations with 12 unknowns. To solve for the equilibrium we use recursive substitution and after standard algebra obtain the expressions for A_D, A_F and B_j in the proposition. In detail, to derive the expression for B_D we proceed as follows. Substituting (A.3) in (A.4) and rearranging we obtain:

$$b_D^I = d\gamma_D \frac{1}{d^2 B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2)}. \tag{A.16}$$

Replacing (A.15), (A.16), and (A.3) in (A.10) and simplifying yields

$$b_D^O = a_D^O \frac{d^2\mu_D\gamma_D(d^2B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2))}{B_D(\mu_D^2 d^2\gamma_D^2(1 + B_F^2\sigma_{u_F}^2) + \sigma_{u_D}^2(d^2B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2))^2)}. \tag{A.17}$$

Similarly, replacing (A.16) in (A.9) and simplifying yields

$$\begin{aligned} a_D^O &= \gamma_D \frac{\mu_D^2 d^2\gamma_D^2(1 + B_F^2\sigma_{u_F}^2) + \sigma_{u_D}^2(d^2B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2))^2}{(d^2B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2))(\mu_D^2 d^2\gamma_D^2 + \sigma_{u_D}^2(\sigma_\eta^2 + d^2)(\sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2) + d^2B_F^2\sigma_{u_F}^2))} \\ &= \gamma_D \frac{\mu_D^2\gamma_D^2\rho_D^2 + \sigma_{u_D}^2(1 - \rho_D^2)^2}{\text{Var}[v_D|\delta_D](1 - \rho_D^2)(\mu_D^2\gamma_D^2\rho_D^2 + \sigma_{u_D}^2\text{Var}[v_D|\delta_D](1 - \rho_D^2))}. \end{aligned} \tag{A.18}$$

Using (A.18) to simplify (A.17) yields

$$b_D^O = \gamma_D^2 \frac{d^2 \mu_D}{B_D(\mu_D^2 d^2 \gamma_D^2 + \sigma_{u_D}^2(\sigma_\eta^2 + d^2)(\sigma_\eta^2(1 + B_F^2 \sigma_{u_F}^2) + d^2 B_F^2 \sigma_{u_F}^2))}. \tag{A.19}$$

We can now replace (A.3), (A.18) and (A.19) in (A.14) and after some tedious algebra obtain

$$B_D = \frac{(\sigma_{u_D}^2(\sigma_\eta^2 + d^2)(\sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2) + d^2 B_F^2 \sigma_{u_F}^2) + \mu_D d^2 \gamma_D^2)(d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2 \sigma_{u_F}^2))}{\gamma_D(\mu_D^2 d^2 \gamma_D^2(1 + B_F^2 \sigma_{u_F}^2) + \sigma_{u_D}^2(\sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2) + d^2 B_F^2 \sigma_{u_F}^2)((1 + B_F^2 \sigma_{u_F}^2)\sigma_\eta^2 + d^2(\mu_D + B_F^2 \sigma_{u_F}^2)))}. \tag{A.20}$$

In a similar fashion we obtain

$$B_F = \frac{(\mu_F \gamma_F^2 + B_D^2 \sigma_{u_F}^2 \sigma_{u_D}^2) B_D^2 \sigma_{u_D}^2}{\gamma_F(\mu_F(\mu_F \gamma_F^2 + B_D^2 \sigma_{u_F}^2 \sigma_{u_D}^2) + B_D^2 \sigma_{u_D}^2(\mu_F^2 \gamma_F^2 + B_D^2 \sigma_{u_F}^2 \sigma_{u_D}^2))}. \tag{A.21}$$

To obtain the expressions displayed in proposition 2 recall that according to our definitions

$$\rho_D^2 = \frac{d^2}{(\sigma_\eta^2 + d^2)(1 + B_F^2 \sigma_{u_F}^2)}.$$

Using this definition to rearrange the numerator of (A.20) yields

$$(1 + B_F^2 \sigma_{u_F}^2)^2 (\sigma_\eta^2 + d^2)^2 (\mu_D \gamma_D^2 \rho_D^2 + (\sigma_\eta^2 + d^2)\sigma_{u_D}^2(1 - \rho_D^2)). \tag{A.22}$$

Similarly, the denominator of (A.20) can be expressed as follows

$$\gamma_D(1 + B_F^2 \sigma_{u_F})^2 (\sigma_\eta^2 + d^2)(\rho_D^2 \mu_D^2 \gamma_D^2 + \sigma_{u_D}^2(\sigma_\eta^2 + d^2)(1 - \rho_D^2)(1 - \rho_D^2(1 - \mu_D))). \tag{A.23}$$

Finally, note that $B_{D0} = (\sigma_\eta^2 + d^2)/\gamma_D$. Hence, using (A.22) and (A.23) we obtain

$$B_D = B_{D0}(1 - \rho_D^2)\left(\frac{\mu_D \gamma_D^2 \rho_D^2 + (\sigma_\eta^2 + d^2)\sigma_{u_D}^2(1 - \rho_D^2)}{\rho_D^2 \mu_D^2 \gamma_D^2 + \sigma_{u_D}^2(\sigma_\eta^2 + d^2)(1 - \rho_D^2)(1 - \rho_D^2(1 - \mu_D))}\right). \tag{A.24}$$

Similarly, we can rearrange (A.21) making use of

$$\rho_F^2 = \frac{1}{1 + B_D^2 \sigma_{u_D}^2},$$

and obtain

$$B_F = B_{F0}(1 - \rho_F^2)\left(\frac{\mu_F \gamma_F^2 \rho_F^2 + \sigma_{u_F}^2(1 - \rho_F^2)}{\rho_F^2 \mu_F^2 \gamma_F^2 + \sigma_{u_F}^2(1 - \rho_F^2)(1 - \rho_F^2(1 - \mu_F))}\right). \tag{A.25}$$

Given that $\mathrm{Var}[v_D|\delta_D] = \sigma_\eta^2 + d^2$, while $\mathrm{Var}[v_F|\delta_F] = 1$, this completes this part of the proof.

Step 2. Existence when $\mu_j = 1$.

When $\mu_D = \mu_F = 1$, a rational expectations equilibrium exists if the following system of equations has a solution (see the discussion at the end of Section 4.2):

$$B_{D1} = f_1(B_{F1}; \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) = \frac{\sigma_\eta^2}{\gamma_D} + \frac{d^2 B_{F1}^2 \sigma_{u_F}^2}{\gamma_D(1 + B_{F1}^2 \sigma_{u_F}^2)}, \tag{A.26}$$

$$B_{F1} = g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2) = \frac{B_{D1}^2 \sigma_{u_D}^2}{\gamma_F(1 + B_{D1}^2 \sigma_{u_D}^2)}. \tag{A.27}$$

Substituting the expression for B_{F1} in equation (A.26) and rearranging, we obtain that the equilibrium level of illiquidity for security 1 solves:

$$\Psi(B_{D1}) \stackrel{def}{=} \left(\sigma_\eta^2 - \gamma_D B_{D1}\right)\left(\gamma_F^2(1 + B_{D1}^2\sigma_{u_D}^2)^2 + B_{D1}^4\sigma_{u_D}^4\sigma_{u_F}^2\right) + d^2 B_{D1}^4\sigma_{u_D}^4\sigma_{u_F}^2 = 0, \quad \text{(A.28)}$$

which is a quintic in B_{D1}. Observe that

$$\Psi\left(\frac{\sigma_\eta^2}{\gamma_D}\right) \geq 0, \ \ \Psi\left(\frac{\sigma_\eta^2 + d^2}{\gamma_D}\right) < 0,$$

and $\Psi(\cdot)$ is continuous. Thus, (A.28) has at least one solution B_{D1}^* in the interval $[\sigma_\eta^2/\gamma_D, (\sigma_\eta^2 + d^2)/\gamma_D]$. This proves existence of a noisy equilibrium when $\mu_D = \mu_F = 1$. Furthermore, $\Psi(0) = \gamma_F^2\sigma_\eta^2 > 0$, which implies that there is no fully revealing equilibrium as long as $\sigma_\eta > 0$.

Step 3. Existence when $\mu_j < 1$.

With limited attention the system of equations that determines the illiquidity of the two markets is highly nonlinear. In this case, substituting (A.21) in (A.20) yields an odd-degree polynomial in B_D with negative leading coefficient: $\Psi(B_D) \equiv f(B_F(B_D)) - B_D$. Hence,

$$\lim_{B_D \to \infty} \Psi(B_D) = -\infty,$$

while

$$\Psi(0) = \gamma_F^{12}\mu_F^8\sigma_\eta^2(d^2\gamma_D^2\mu_D + \sigma_\eta^2\sigma_{u_D}^2(d^2 + \sigma_\eta^2)) > 0,$$

which implies that there always exists a strictly positive value B_D^*, such that $\Psi(B_D^*) = 0$.

□

Proof of Corollary 1

For a unique equilibrium to obtain, we need $\Psi'(B_{D1}) < 0$, $\forall B_{D1}$. Computing the derivative of $\Psi(B_{D1})$ yields:

$$\Psi'(B_{D1}) = -\gamma_D\gamma_F^2(1 + B_{D1}^2\sigma_{u_D}^2)^2 + \\ 4B_{D1}\sigma_{u_D}^2(\sigma_\eta^2 - \gamma_D B_{D1})\left(\gamma_F^2(1 + B_{D1}^2\sigma_{u_D}^2) + B_{D1}^2\sigma_{u_D}^2\sigma_{u_F}^2\right) + B_{D1}^3\sigma_{u_D}^4\sigma_{u_F}^2\gamma_D(4\gamma_D^{-1}d^2 - B_{D1}).$$

At any equilibrium the first two terms in the expression above are negative, (the second term is negative, since at any equilibrium $B_{D1} > \sigma_\eta^2/\gamma_D$). To ensure that the last term is also negative, we thus impose $4d^2/\gamma_D \leq \sigma_\eta^2/\gamma_D$.

□

Proof of Corollary 3

The result follows immediately from equations (4.9) and (4.10). □

Proof of Corollary 5

To prove our claim, note that owing to our definitions the equilibrium value for the illiquidity in market D obtains as a solution to the system (A.26)–(A.27) or, equivalently, as a solution to the equation

$$\begin{aligned} B_{D1} &= f_1(g_1(B_{D1}); \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) \\ &= \frac{\sigma_\eta^2}{\gamma_D} + \frac{d^2(g_1(B_{D1}))^2\sigma_{u_F}^2}{\gamma_D(1+(g_1(B_{D1}))^2\sigma_{u_F}^2)}. \end{aligned}$$

Therefore, at equilibrium it must be that

$$\frac{\partial f_1}{\partial B_{F1}}\frac{\partial g_1}{\partial B_{D1}} \equiv \frac{\partial f_1}{\partial B_{D1}}.$$

Hence, to verify that at equilibrium $\kappa > 1$ it is necessary and sufficient to verify that $f_1'(B_{D1}) < 1$ at equilibrium. Rearranging (A.28) shows that equilibria obtain as a solution to the following quintic:

$$\begin{aligned} \Psi(B_{D1}) = f_1(B_{D1}) - B_{D1} = &-B_{D1}^5\gamma_D\sigma_{u_D}^4(\gamma_F^2+\sigma_{u_F}^2) + B_{D1}^4\sigma_{u_D}^4(\gamma_F^2\sigma_\eta^2 + (d^2+\sigma_\eta^2)\sigma_{u_F}^2) \\ &- 2B_{D1}^3\gamma_D\gamma_F^2\sigma_{u_D}^2 + 2B_{D1}\gamma_F^2\sigma_\eta^2\sigma_{u_D}^2 - B_{D1}\gamma_D\gamma_F^2 + \gamma_F^2\sigma_\eta^2, \end{aligned}$$

which owing to Descartes' rule of signs possesses 5, 3 or 1 positive root. These roots correspond to the intersections of the function $f_1(B_{D1})$ with the 45-degree line. Given that $f_1(0) = \sigma_\eta^2/\gamma_D > 0$ and,

$$f_1'(B_{D1}) = \frac{4B_D^3 d^2\gamma_F^2\sigma_{u_D}^4\sigma_{u_F}^2(1+B_D^2\sigma_{u_D}^2)}{\gamma_D(\gamma_F^2(1+B_D^2\sigma_{u_D}^2)^2 + B_D^4\sigma_{u_D}^4\sigma_{u_F}^2)^2} > 0,$$

the function $f_1(B_{D1})$ cuts for the first time the 45-degree line from above, and thus $\kappa > 1$ in this case. Hence, there always exists an equilibrium in which $f'(B_{D1}) < 1$ at the intersection with the 45-degree line.

When the equilibrium is unique, this is the only equilibrium that survives. To see this, note that

$$f_1''(B_{D1}) = -\frac{4B_D^2 d^2\gamma_F^2\sigma_{u_D}^4\sigma_{u_F}^2(3\gamma_F^2(-1+B_D^2\sigma_{u_D}^2)(1+B_{D1}^2\sigma_{u_D}^2)^2 + B_{D1}^4\sigma_{u_D}^4\sigma_{u_F}^2(5+3B_{D1}^2\sigma_{u_D}^2))}{\gamma_D(\gamma_F^2(1+B_D^2\sigma_{u_D}^2)^2 + B_D^4\sigma_{u_D}^4\sigma_{u_F}^2)^3},$$

so that $f_1(B_{D1})$ has as many changes in curvature as the number of roots of the polynomial at the numerator of $f''(B_{D1})$. Expanding this numerator yields

$$-3B_{D1}^6\sigma_{u_D}^6(\gamma_F^2+\sigma_{u_F}^2) - B_{D1}^4\sigma_{u_D}^4(3\gamma_F^2+5\sigma_{u_F}^2) + 3B_{D1}^2\gamma_F^2\sigma_{u_D}^2 + 3\gamma_F^2,$$

which making the substitution $B_{D1}^2 = y$ can be rewritten as follows

$$-3y^3\sigma_{u_D}^6(\gamma_F^2+\sigma_{u_F}^2) - y^2\sigma_{u_D}^4(3\gamma_F^2+5\sigma_{u_F}^2) + 3y\gamma_F^2\sigma_{u_D}^2 + 3\gamma_F^2, \tag{A.29}$$

a cubic in y which (again due to Descartes rule of signs) has only one positive root. Note also that for $y = 0$ the above polynomial is positive, implying that f_1 is convex for $B_{D1} = 0$. This has two implications: (1) there can be at most 3 equilibria (since to have 5 equilibria we would need two changes of curvature for $f_1(\cdot)$, but this requires three real roots for the numerator of f_1'' which we do not get as said above) and (2) that multiple equilibria occur when f_1 cuts the 45-degree the first time when it is convex (i.e., the second and third equilibria occur with f_1 respectively convex and concave), so that the three equilibria feature $\kappa > 1$, $\kappa < 1$, and $\kappa > 1$ respectively. (1) and (2) in turn imply that whenever the equilibrium is unique, it must be that $\kappa > 1$. Rearranging (A.29) in the following way:

$$3y\sigma_{u_D}^2(\gamma_F^2 - y^2\sigma_{u_D}^4(\gamma_F^2 + \sigma_{u_F}^2)) + (3\gamma_F^2 - y^2\sigma_{u_D}^4(3\gamma_F^2 + 5\sigma_{u_F}^2)), \tag{A.30}$$

we can find an interval that binds its root. Indeed, it is easy to see that

$$\frac{\gamma_F^2}{\sigma_{u_D}^4(\gamma_F^2 + \sigma_{u_F}^2)} > \frac{3\gamma_F^2}{\sigma_{u_D}^4(3\gamma_F^2 + 5\sigma_{u_F}^2)}.$$

Therefore, for

$$y \in \left(\frac{\gamma_F}{\sigma_{u_D}^2} \left(\frac{3}{(3\gamma_F^2 + 5\sigma_{u_F}^2)} \right)^{1/2}, \frac{\gamma_F}{\sigma_{u_D}^2(\gamma_F^2 + \sigma_{u_F}^2)^{1/2}} \right),$$

the first term in (A.30) is positive while the second one is negative. Hence, the change of curvature in $f_1(\cdot)$ must occur for

$$y \in \left(\frac{\gamma_F}{\sigma_{u_D}^2} \left(\frac{3}{(3\gamma_F^2 + 5\sigma_{u_F}^2)} \right)^{1/2}, \frac{\gamma_F}{\sigma_{u_D}^2(\gamma_F^2 + \sigma_{u_F}^2)^{1/2}} \right),$$

or for

$$B_{D1} \in \left(\left(\frac{\gamma_F}{\sigma_{u_D}^2} \left(\frac{3}{(3\gamma_F^2 + 5\sigma_{u_F}^2)} \right)^{1/2} \right)^{1/2}, \left(\frac{\gamma_F}{\sigma_{u_D}^2(\gamma_F^2 + \sigma_{u_F}^2)^{1/2}} \right)^{1/2} \right).$$

So, if we impose that

$$f_1(0) = \frac{\sigma_\eta^2}{\gamma_D} > \left(\frac{\gamma_F}{\sigma_{u_D}^2(\gamma_F^2 + \sigma_{u_F}^2)^{1/2}} \right)^{1/2},$$

then we can be sure that the change of curvature occurs for values of f_1 "above" the 45-degree line, implying that $\Psi(B_{D1})$ has a unique real root (incidentally, this provides an alternative sufficient condition for uniqueness that also rationalizes our numerical examples – see below). Given that this root occurs when f_1 is concave, this also implies that at this equilibrium $\kappa > 1$. To double check (this is of course just one numerical example) note that with the values we use in figure 2 we obtain

$$f_1(0) = \frac{\sigma_\eta^2}{\gamma_D} \equiv 1 > \left(\frac{\gamma_F}{\sigma_{u_D}^2(\gamma_F^2 + \sigma_{u_F}^2)^{1/2}} \right)^{1/2} \equiv \sqrt{\frac{1}{4}\sqrt{\frac{1}{5}}}.$$

For a graphical illustration of the argument of this proof see Figure 12.

Part 1 of the corollary follows immediately since, as explained in the text, we have $(dB_{D1}/d\gamma_D) < 0$ and $(dB_{F1}/d\gamma_D) < 0$ iff $\kappa > 1$. The effects of a change in γ_F and σ_η^2 can be analized in the same way. $\square$

Proof of Corollary 6

The first part of the corollary follows immediately by substituting $d = 0$ in the expressions for illiquidity obtained in Proposition 2. For the second part, we differentiate B_F with respect to B_D and verify that

$$\frac{\partial B_F}{\partial B_D} = \frac{2B_{D\mu_F\sigma^2_{u_D}}(\gamma_F^4\mu_F^2 + B_D^2\gamma_F^2\sigma^2_{u_D}\sigma^2_{u_F}(2\mu_F - B_D^2(1-\mu_F)\sigma^2_{u_D}) + B_D^4\sigma^4_{u_D}\sigma^4_{u_F})}{\gamma_F((\gamma_F\mu_F)^2(1+B_D^2\sigma^2_{u_D}) + B_D^2\sigma^2_{u_D}\sigma^2_{u_F}(\mu_F + B_D^2\sigma^2_{u_D}))^2}. \tag{A.31}$$

The numerator of the above expression contains a quadratic in μ_F with positive leading coefficient. Hence, its sign is positive for all values of μ_F which are in absolute value larger than the two real roots that solve the equation. Upon inspection, the first of these roots is always negative, whereas the other root is given by

$$\hat{\mu}_F = \frac{B_D^2\sigma^2_{u_D}\sigma_{u_F}\left(-(2+B_D^2\sigma^2_{u_D})\sigma_{u_F} + \sqrt{4\gamma_F^2 + B_D^2\sigma^2_{u_D}\sigma^2_{u_F}(4+B_D^2\sigma^2_{u_D})}\right)}{2\gamma_F^2},$$

which is positive if and only if $\gamma_F^2 > \sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F]$. In this case, whenever $\mu_F > \hat{\mu}_F$, (A.31) is positive. The last part follows in a similar fashion. Differentiating B_F with respect to μ_F yields

$$\frac{\partial B_F}{\partial \mu_F} = -\frac{B_D^2\sigma^2_{u_D}(\gamma_F^4\mu_F^2 - B_D^4\sigma^4_{u_D}\sigma^2_{u_F}(\gamma_F^2(1-2\mu_F) - \sigma^2_{u_F}) + B_D^2\gamma_F^2\mu_F\sigma^2_{u_D}(\gamma_F^2\mu_F + 2\sigma^2_{u_F}))}{\gamma_F((\gamma_F\mu_F)^2(1+B_D^2\sigma^2_{u_D}) + B_D^2\sigma^2_{u_D}\sigma^2_{u_F}(\mu_F + B_D^2\sigma^2_{u_D}))^2}. \tag{A.32}$$

The numerator of the above expression contains a quadratic in μ_F with positive leading coefficient. Hence, its sign is positive for all values of μ_F which are in absolute value larger than the two real roots that solve the equation. Upon inspection, the first of these roots is always negative, whereas the other root is given by

$$\hat{\hat{\mu}}_F = \frac{-B_D^2\sigma^2_{u_D}\sigma_{u_F}\left(\sigma_{u_F}(1+B_D^2\sigma^2_{u_D}) - ((1+B_D^2\sigma^2_{u_D})(\gamma_F^2 + B_D^2\sigma^2_{u_D}\sigma^2_{u_F}))^{1/2}\right)}{\gamma_F^2(1+B_D^2\sigma^2_{u_D})},$$

which is positive if and only if $\gamma_F^2 > \sigma^2_{u_F}\mathrm{Var}[v_F|\delta_F]$. In this case, whenever $\mu_F > \hat{\hat{\mu}}_F$, (A.32) is negative.

$\square$

Proof of Corollary 7

Let us define $G(\mu_j, \rho_j^2)$ as:

$$G(\mu_j, \rho_j^2) = \frac{\gamma_j^2\mu_j\rho_j^2 + \sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j](1-\rho_j^2)}{\gamma_j^2\mu_j^2\rho_j^2 + \sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j](1-\rho_j^2)(1-\rho_j^2(1-\mu_j))},$$

so that $B_j = B_{j0}(1-\rho_j^2)G(\mu_j, \rho_j^2)$. Now observe that:

$$\frac{\partial G(\mu_D, \rho_D^2)}{\partial \rho_D^2} = \frac{(\sigma_\eta^2 + d^2)(1-\mu_D)(1-\rho_{_D}^2)\sigma^2_{u_D}(\gamma^2\mu_D(1+\rho_{_D}^2) + (\sigma_\eta^2+d^2)(1-\rho_{_D}^2)\sigma^2_{u_D})}{(\gamma^2\mu_D^2\rho_{_D}^2 + \sigma^2_{u_D}\mathrm{Var}[v_D|\delta_D](1-\rho_D^2)(1-\rho_D^2(1-\mu_D)))^2} > 0.$$

Therefore, we have:
$$\frac{\partial B_D}{\partial \rho^2_{_D}} = -B_{0D}G(\mu_D, \rho^2_D) + B_{0D}(1-\rho^2_D)\frac{\partial G}{\partial \rho^2_D},$$
which yields
$$\frac{\partial B_D}{\partial \rho^2_{_D}} = -\frac{\mathrm{Var}[v_D|\delta_D]\mu_D}{\gamma(\gamma^2\mu^2_D\rho^2_{_D} + \sigma^2_u \mathrm{Var}[v_D|\delta_D, \omega_F](1-\rho^2_{_D}(1-\mu_D)))^2}\times$$
$$(\gamma^4\mu^2_D\rho^4_{_D} + \sigma^2_{u_D}\mathrm{Var}[v_D|\delta_D](1-\rho^2_D)(\mathrm{Var}[v_D|\delta_D](1-\rho^2_{_D})\sigma^2_u - \gamma^2(1-\mu_D-\rho^2_D(1+\mu_D)))).$$
Since
$$\frac{1-\rho^2_D}{1+\rho^2_D} < 1,$$
if
$$\mu_D > \underbrace{1 - \frac{\mathrm{Var}[v_D|\delta_D]\sigma^2_{u_D}}{\gamma^2_D}}_{\overline{\mu}_D},$$
then $(\partial B_D/\partial \rho^2_{_D}) < 0$. As $(\partial \rho^2_{_D}/\partial B_F) < 0$, and B_F affects B_D only through its effect on $\rho^2_{_D}$, we deduce that $(\partial f/\partial B_F) > 0$ if $\mu_D > \overline{\mu}_D$. A similar reasoning shows that $(\partial g/\partial B_D) > 0$ if $\mu_F > \overline{\mu}_F$.

$\square$

Proof of Corollary 8

Computing the direct effect of a change in attention in market j on the illiquidity of the same market yields:
$$\frac{\partial B_j}{\partial \mu_j} = B_{0j}(1-\rho^2_j)\frac{\partial G(\mu_j, \rho^2_j)}{\partial \mu_j}, \tag{A.33}$$
where $G(\mu_j, \rho^2_j)$ is defined in the proof of Corollary 7, and
$$\frac{\partial G(\mu_j, \rho^2_j)}{\partial \mu_j} = \frac{-\gamma^4_j\mu^2_j\rho^4_j + \sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j](1-\rho^2_j)\rho^2_j(-2\mu_j\gamma^2_j + (1-\rho^2_j)(\gamma^2_j - \sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j]))}{(\gamma^2_j\mu^2_j\rho^2_j + \sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j](1-\rho^2_j)(1-\rho^2_j(1-\mu_j)))^2}.$$
Inspection of the numerator in the above expression shows that if
$$\sigma^2_{u_j}\mathrm{Var}[v_j|\delta_j] > \gamma^2_j, \tag{A.34}$$
then
$$\frac{\partial G(\mu_j, \rho^2_j)}{\partial \mu_j} < 0.$$
Thus, if (A.34) holds, the direct effect of an increase in μ_j is to increase the liquidity of market j. As the discussion in Section 4.2 has clarified, with bi-directional liquidity spillovers, the direct effect is part of the total effect that a change in a liquidity fundamental determines on the liquidity of both markets. The same argument here implies that for the direct effect of an increase in attention in security j to deliver a reduction in B_j and B_{-j} we need to make sure that at equilibrium $\kappa > 1$. The rest of the proof thus shows that there is always an equilibrium in which this occurs.

To prove that there always exist an equilibrium in which $\kappa > 1$ we use the same argument adopted in the proof of Corollary 5. If we substitute (A.21) into (A.20), the equilibrium obtains as a solution of the equation $B_D - f(g(B_D)) = 0$. Equivalently, the equilibrium obtains as a fixed point of

$$B_D = f(g(B_D)).$$

Note that

$$f(g(B_D))|_{B_D=0} = \frac{\sigma_\eta^2(d^2\gamma_D^2\mu_D + \sigma_\eta^2\sigma_{u_D}^2(d^2+\sigma_\eta^2))}{\gamma_D(\sigma_\eta^4\sigma_{u_D}^2 + d^2\mu_D(\gamma_D^2\mu_D + \sigma_\eta^2\sigma_{u_D}^2)} > 0, \tag{A.35}$$

and

$$f'(B_D) =$$

$$\frac{1}{(\gamma_D(\mu_D^2 d^2\gamma_D^2(1+B_F^2\sigma_{u_F}^2) + \sigma_{u_D}^2(\sigma_\eta^2(1+B_F^2\sigma_{u_F}^2) + d^2B_F^2\sigma_{u_F}^2)((1+B_F^2\sigma_{u_F}^2)\sigma_\eta^2 + d^2(\mu_D + B_F^2\sigma_{u_F}^2))))^2} \times$$
$$\Big(4B_DB_Fd^2\mu_D\mu_F\sigma_{u_D}^2\sigma_{u_F}^2\,\big(\gamma_F^4\mu_F^2 + B_D^2\gamma_F^2\sigma_{u_D}^2\sigma_{u_F}^2(2\mu_F + B_D^2\sigma_{u_D}^2(\mu_F-1)) + B_D^4\sigma_{u_D}^4\sigma_{u_F}^4\big) \times$$
$$(d^4\gamma_D^4\mu_D^2 + \gamma_D^2\sigma_\eta^2\sigma_{u_D}^2(d^2+\sigma_\eta^2)(2d^2\mu_D + (\mu_D-1)\sigma_\eta^2) + (d^2+\sigma_\eta^2)^2\sigma_{u_D}^2(\sigma_\eta^4\sigma_{u_D}^2 + \sigma_{u_F}^2B_F^2(2\gamma_D^2(d^2\mu_D + (\mu_D - 1$$
$$+2\sigma_\eta^2\sigma_{u_D}^2(d^2+\sigma_\eta^2) + \sigma_{u_F}^2B_F^2(d^2+\sigma_\eta^2)(\gamma_D^2(\mu_D-1) + (d^2+\sigma_\eta^2)\sigma_{u_D}^2)))\Big).$$

The above expression is strictly increasing in μ_D and μ_F and is null for $\mu_D = \mu_F = 0$. This implies that for all positive μ_D, μ_F, $f'(B_D) > 0$. Also, owing to (A.35), $f(g(B_D))$ has a positive intercept (i.e., is "above" the 45-degree line for $B_D = 0$). Therefore, there must exist at least one value of B_D, B_D^* such that $f(B_D^*) = B_D^*$, and $f'(B_D^*) < 1$, and hence $\kappa > 1$. For this equilibrium we have that the direct effect determines the sign of (A.33), and the result follows.

$\square$

Proof of Proposition 3

Using the notations introduced in the proof of Proposition 2, we have:

$$\mathrm{Var}[v_F|\delta_F, \hat{\omega}_F] = \gamma_F(a_F^O)^{-1}$$
$$\mathrm{Var}[v_F|\delta_F, \omega_F] = \gamma_F(a_F^I)^{-1},$$

where

$$a_F^I = \gamma_F\frac{1+B_D^2\sigma_{u_D}^2}{B_D^2\sigma_{u_D}^2}, \quad a_F^O = \gamma_F\frac{\mu_F^2\gamma_F^2(1+B_D^2\sigma_{u_D}^2) + B_D^4\sigma_{u_D}^4\sigma_{u_F}^2}{B_D^2\sigma_{u_D}^2(\mu_F^2\gamma_F^2 + B_D^2\sigma_{u_D}^2\sigma_{u_F}^2)}.$$

We deduce that:

$$\phi_F(\mu_F, B_D) = \frac{\gamma_F}{2}\ln\left(\frac{a_F^I}{a_F^O}\right),$$

and the expression for $\phi_F(\mu_F, B_D)$ given in the corollary follows. It is then immediate that $\partial\phi_F(\mu_F)/\partial\mu_F < 0$. $\square$

Proof of Proposition 4

As explained in the text, the fraction of pricewatchers in equilibrium is nil if $\phi_F(0, B_D) < C$. Using equation (5.3), we deduce that this condition is satisfied iff $C > \overline{C}$ where:

$$\overline{C} = \frac{\gamma_F}{2}\ln\left(1 + \frac{1}{\sigma_{u_D}^2 B_D^2}\right)$$

Similarly, the fraction of pricewatchers in equilibrium is one iff $C < \underline{C}$ where:

$$\underline{C} = \frac{\gamma_F}{2} \ln\left(1 + \frac{\sigma_{u_F}^2 \sigma_{u_D}^2 B_D^2}{\gamma_F^2(1 + B_D^2 \sigma_{u_D}^2) + \sigma_{u_F}^2 \sigma_{u_D}^4 B_D^4}\right)$$

Otherwise the fraction of pricewatchers in equilibrium solves $\phi_F(\mu_F, B_D) = C$ and we obtain the expression for μ_F^* by inverting ϕ_F. □

Proof of Corollary 9

For a given value of C, the level of illiquidity of security F is given by $B_F(\mu_F^*(C))$ where $B_F(\cdot)$ is given in equation (4.29) when $d = 0$. Thus:

$$\frac{\partial B_F}{\partial C} = \left.\frac{\partial B_F}{\partial \mu_F}\right|_{\mu_F = \mu_F^*(C)} \left(\frac{\partial \mu_F^*(C)}{\partial C}\right).$$

We know that $(\partial \mu_F^*(C)/\partial C) \leq 0$ (Proposition 4). Moreover, using equation (4.29), we deduce that when $d = 0$, $(\partial B_F/\partial \mu_F) < 0$ if and only if $\mu_F > \widehat{\mu}_F$ where

$$\widehat{\mu}_F = \left(\frac{\sigma_\eta^4 \sigma_{u_D}^2 \sigma_{u_F}}{\gamma_F}\right) \sqrt{\frac{\max\{\gamma_F^2 - \sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F], 0\}}{\gamma_D^2 + \sigma_\eta^4 \sigma_{u_D}^2}}.$$

Thus, when $\gamma_F^2 \leq \sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F]$, $\widehat{\mu}_F = 0$ and $(\partial B_F/\partial \mu_F)|_{\mu_F = \mu_F^*(C)} < 0$. It follows that $(\partial B_F/\partial C) > 0$. When $\gamma_F^2 > \sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F]$ then $\widehat{\mu}_F > 0$. As $\mu_F^*(C)$ decreases with C from one to zero over $[\underline{C}, \overline{C}]$, there exists a value $C^* \in (\underline{C}, \overline{C})$ such $\mu_F^*(C) = \widehat{\mu}_F$ and $\mu_F^*(C) < \widehat{\mu}_F$ iff $C > C^*$. Thus, in this case, $(\partial B_F/\partial \mu_F) < 0$ iff $C < C^*$. The second part of the corollary follows. □

Proof of Proposition 5

We have

$$\phi_j(1, B_{j1}^H) = \frac{\gamma}{2} \ln\left(1 + \frac{(B_{j1}^H)^2 \sigma_u^4}{\gamma^2(1 + (B_{j1}^H)^2 \sigma_u^2) + (B_{j1}^H)^4 \sigma_u^6}\right), \tag{A.36}$$

and

$$\phi_j(0, B_{j0}) = \frac{\gamma}{2} \ln\left(1 + \frac{\sigma_\delta^2}{B_{j0}^2 \sigma_u^2}\right) = \frac{\gamma}{2} \ln\left(1 + \frac{\gamma^2}{\sigma_u^2}\right)$$

Thus,

$$\phi_j(1, B_{j1}^H) > \phi_j(0, B_{j0}) \Leftrightarrow \frac{(B_{j1}^H)^2 \sigma_u^4}{\gamma^2(1 + (B_{j1}^H)^2 \sigma_u^2) + (B_{j1}^H)^4 \sigma_u^6} > \frac{\gamma^2}{\sigma_u^2}. \tag{A.37}$$

We deduce that $\phi_j(1, B_{j1}^H) > \phi_j(0, B^*(0))$ if and only if

$$-\gamma^2 \sigma_u^6 (B_{j1}^H)^4 + (\sigma_u^4 - \gamma^4) \sigma_u^2 (B_{j1}^H)^2 - \gamma^4 > 0. \tag{A.38}$$

Now observe that

$$(B_{j1}^H)^2 = \frac{(B_{j1}^H \sigma_u^2 - \gamma)}{\gamma \sigma_u^2}. \tag{A.39}$$

Thus, we can rewrite condition (A.38) as

$$-\gamma \sigma_u^2 (B_{j1}^H \sigma_u^2 - \gamma)^2 + (\sigma_u^4 - \gamma^4)(B_{j1}^H \sigma_u^2 - \gamma) - \gamma^5 > 0.$$

It can be checked that this inequality holds true if B^H_{j1} belongs to

$$\left(\frac{\gamma}{\sigma_u^2}+\frac{\sigma_u^4-\gamma^4-((\sigma_u^4-\gamma^4)^2-4\gamma^6\sigma_u^2)^{1/2}}{2\gamma\sigma_u^4},\frac{\gamma}{\sigma_u^2}+\frac{\sigma_u^4-\gamma^4+((\sigma_u^4-\gamma^4)^2-4\gamma^6\sigma_u^2)^{1/2}}{2\gamma\sigma_u^4}\right).$$

We now verify that this is the case. First, we check that B^H_{j1} is always larger than the lower bound, that is:

$$\frac{\sigma_u^2+\sigma_u(\sigma_u^2-4\gamma^2)^{1/2}}{2\gamma\sigma_u^2}>\frac{\gamma}{\sigma_u^2}+\frac{\sigma_u^4-\gamma^4-((\sigma_u^4-\gamma^4)^2-4\gamma^6\sigma_u^2)^{1/2}}{2\gamma\sigma_u^4}.$$

Rearranging and simplifying we obtain that the above inequality is satisfied if and only if

$$\sigma_u^3\left(\sigma_u^2-4\gamma^2\right)^{1/2}>\gamma^2\left(2\sigma_u-\gamma^2\right)-\left(\left(\sigma_u-\gamma^4\right)^2-4\gamma^6\sigma_u\right)^{1/2}. \tag{A.40}$$

However, if $\sigma_u^2>4\gamma^2$ (a condition that is required for the equilibrium in the high attention regime to exist) the l.h.s. of (A.40) is positive, while the r.h.s. is negative, and the result follows.[23] Next, we check that B^H_{j1} is always smaller than the upper bound, that is:

$$\frac{\sigma_u^2+\sigma_u(\sigma_u^2-4\gamma^2)^{1/2}}{2\gamma}<\frac{\gamma}{\sigma_u^2}+\frac{\sigma_u^4-\gamma^4+((\sigma_u^4-\gamma^4)^2-4\gamma^6\sigma_u^2)^{1/2}}{2\gamma\sigma_u^4}.$$

Rearranging the above inequality we have

$$\sigma_u^3(\sigma_u^2-4\gamma^2)^{1/2}<\gamma^2(2\sigma_u^2-\gamma^2)+((\sigma_u^4-\gamma^4)^2-4\gamma^6\sigma_u^2)^{1/2}. \tag{A.41}$$

Squaring both sides in the above inequality and rearranging yields

$$-4\gamma^2\sigma_u^6<2\gamma^4\left(\sigma_u^2\left(\sigma_u^2-4\gamma^2\right)+\gamma^8\right)+2\gamma^2\left(2\sigma_u^2-\gamma^2\right)\left(\left(\sigma_u^4-\gamma^4\right)^2-4\gamma^6\sigma_u^2\right)^{1/2}. \tag{A.42}$$

While the l.h.s. of (A.42) is negative, again if $\sigma_u^2>4\gamma^2$, the r.h.s. is positive (since $\sigma_u^2>4\gamma^2$), and the result follows. □

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[23] In particular, the r.h.s. of (A.40) is negative for all $\sigma_u^2>\gamma^2\sqrt{6}$, a condition that is satisfied by the imposed parameter restriction.

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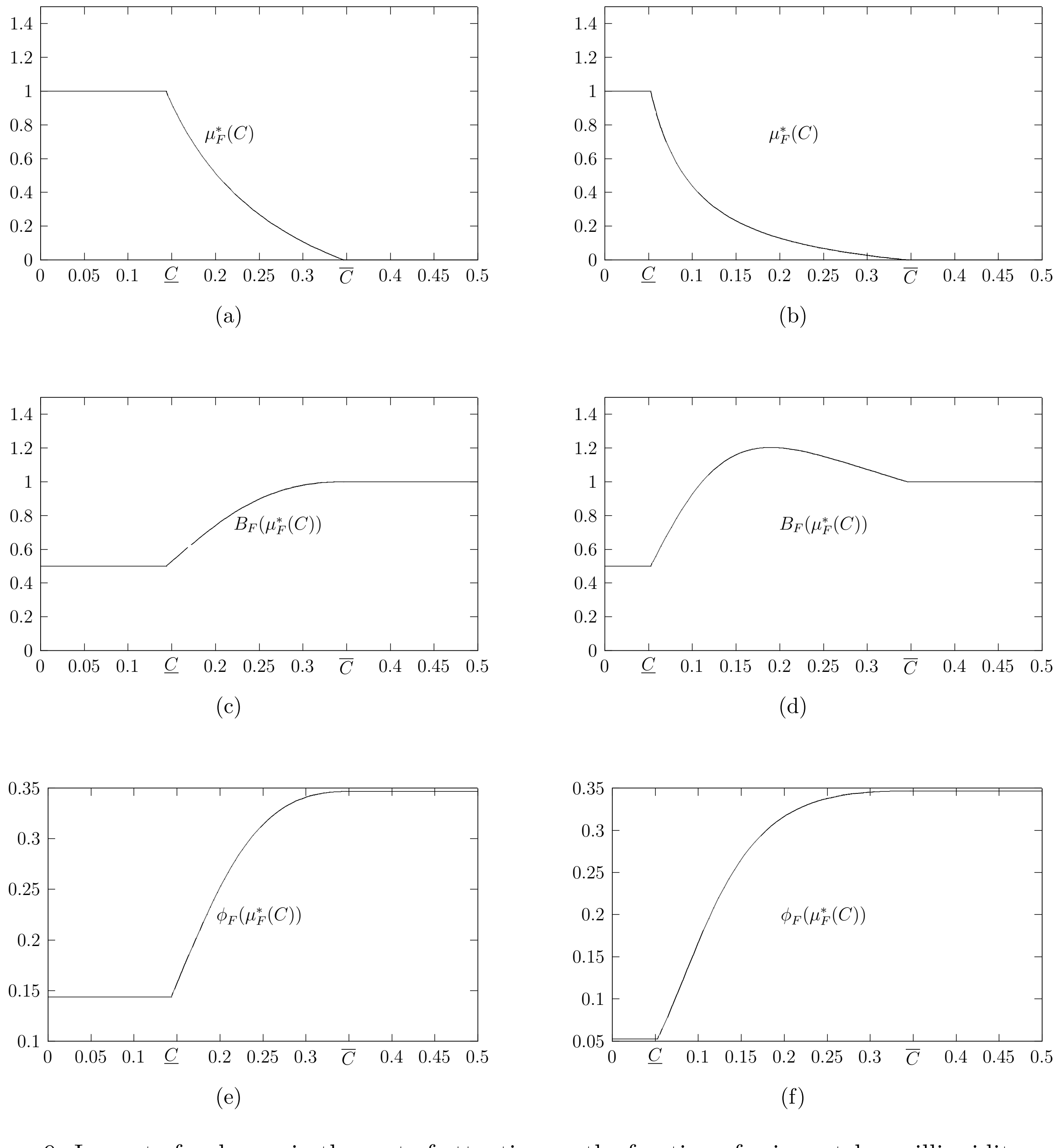

Figure 9: Impact of a change in the cost of attention on the fraction of pricewatchers, illiquidity, and the value of information with one-sided learning. Case with $\sigma^2_{u_F}\text{Var}[v_F|\delta_F] \geq \gamma_F^2$ (panels (a), (c), and (e)), and case with $\sigma^2_{u_F}\text{Var}[v_F|\delta_F] < \gamma_F^2$ (panels (b), (d), and (f)). Parameters' values are as follows: $\sigma_{u_D} = 1$, $\gamma_F = \gamma_D = 1$, $d = 0$, and $\sigma_\eta = 1$, with $\sigma_{u_F} = 1$ in panels (a), (c), and (e) whereas $\sigma_{u_F} = 0.5$ in panels (b), (d), and (f).

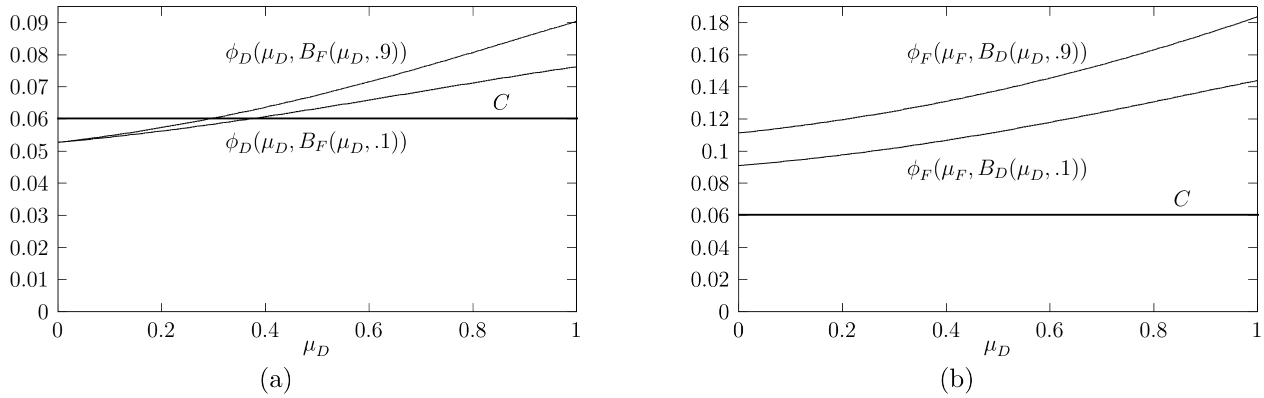

Figure 10: The figure illustrates the relevance of the positive feedback effect on the value of information in the two markets. In panel (a) we plot ϕ_D as a function of μ_D, for $\mu_F \in \{0.1, 0.9\}$. In panel (b) we plot ϕ_F as a function of μ_D, for $\mu_F \in \{0.1, 0.9\}$. Other parameter values are as follows: $\sigma_\eta = 1$, $\sigma_{u_F} = \sigma_{u_D} = 1$, $\gamma_F = \gamma_D = 1$, and $d = 1$.

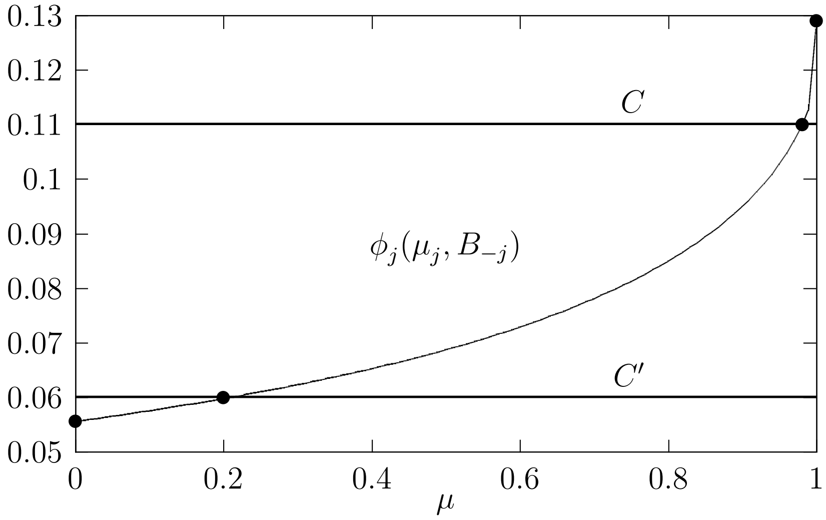

Figure 11: The figure shows that markets can be segmented even if the cost of attention is low. Parameter values are as follows: $\sigma_\delta = \sigma_u = 1$, $\gamma = 1/2$, $d = 1$, and $\sigma_\eta = 0$. When $C = 0.11$ three equilibria arise, with $\mu_j \in \{0, 0.97, 1\}$, the two extreme equilibria survive even if the cost of attention is reduced to $C' = 0.06$, in which case the three equilibria feature $\mu_j \in \{0, 0.2, 1\}$.

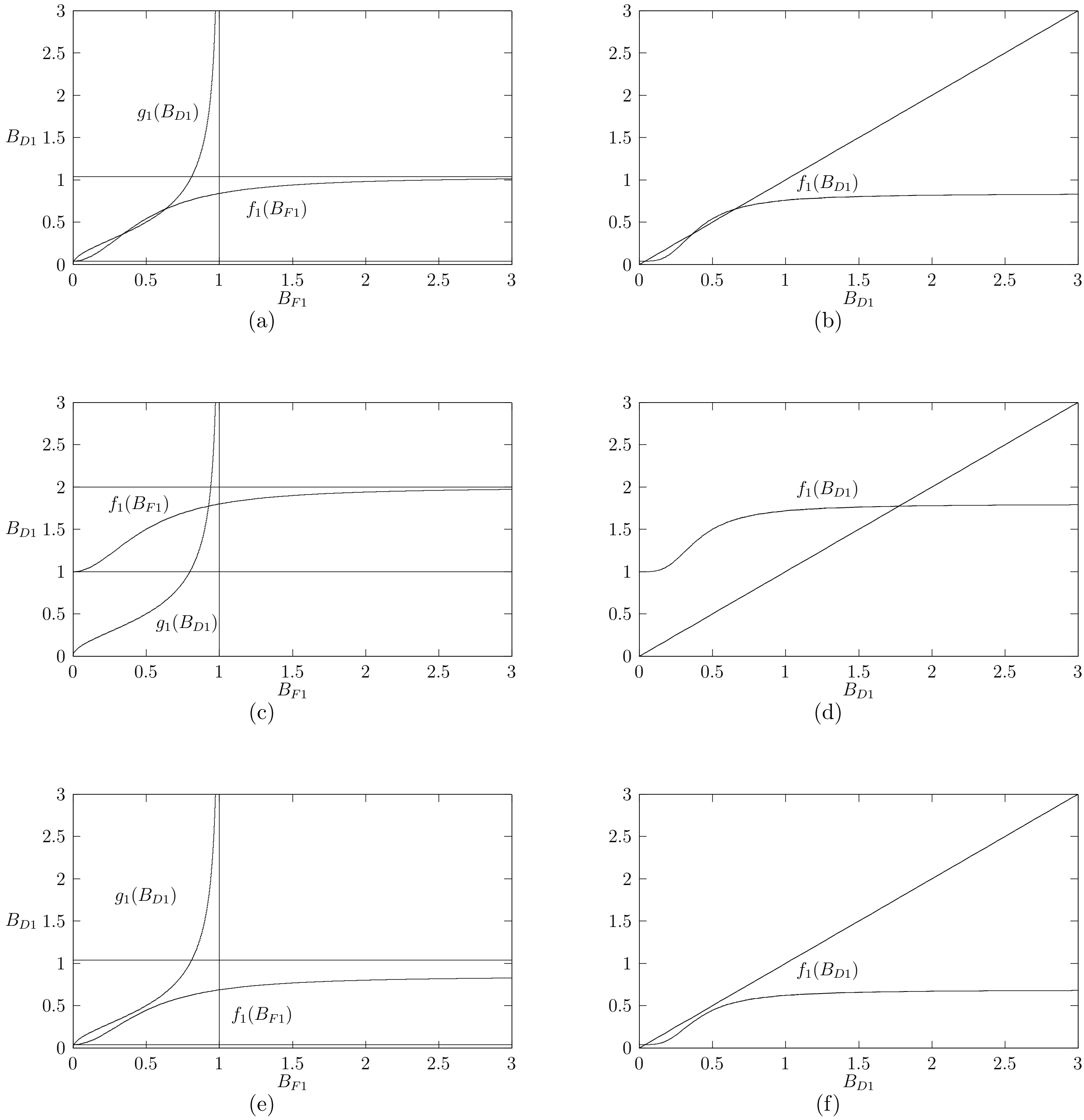

Figure 12: Equilibrium determination with full attention (no adverse selection): multiplicity (panels (a) and (b)) and uniqueness (panels (c)–(f)). Parameters' values are as follows: $\gamma_j = d = 1$, $\sigma_{u_j} = 2$, and $\sigma_\eta = .2$ (panels (a) and (b)), while in panels (c) and (d) we set $\sigma_\eta = 1$ and in panels (e) and (f) we set $d = 0.9$. In the left column the equilibrium obtains via the intersection of the functions $f_1(B_{F1})$ and $g_1(B_{D1})$; in the right column the equilibrium obtains via the intersection of the function $f_1(g_1(B_{D1}))$ with the 45-degree line. Inspection of the unique equilibrium in panels (d) and (f) shows that $f_1'(\cdot) < 1$.