A Model of Market Segmentation with Risk

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Abstract

We characterize an optimal scheme for the sale of multiple identical items by a monopolist in a market comprising risk-averse buyers. We establish that a seller may obtain segmentation benefits by randomizing prices in one channel while also offering a risk-free alternative in another. The optimal vehicle of such randomization is a draw from a discrete two-point probability distribution function. We use the model to offer explanations for observed behavior of online sellers and discuss implementation issues in recent e-commerce environments.

JEL Classifications: D42, L1

1. Introduction

In Internet based commerce, sellers often use multiple distribution channels for the sale of standard consumer goods. For example, within a two week period, Carnival Cruise Lines offered units of the same cabin class on one of its ships in three distinct ways: standard posted prices, ascending bid ("English") auctions and "last minute" clearance sales. A possible explanation for such behavior is that sellers like Carnival are deliberately embedding price uncertainties into their sales channels in order to employ second-degree price discrimination among buyers who are risk-averse. Buyers who assign higher values to the offered product are typically more reluctant to risk compromising their surplus and are therefore prone to purchase at a higher posted price earlier, while buyers with lower values may wait and attempt to acquire the product at a bargain price.

Although we are not aware of any comprehensive field study in this area, there exists much anecdotal evidence to suggest that firms collect significant rents by exploiting buyers' risk aversion. Soberman (2003) reports that about half of the operating profit for large consumer electronic chains comes from selling extended warranties and that margins on extended warranties exceed 70%. In his subsequent analysis, he posits a dual role of warranty contracts in simultaneously signaling high quality and screening buyers and then derives the managerial implications of this duality. Similarly, new questions have arisen regarding the use of auctions and clearance sales in markets. Under what conditions can auctions and random clearance sales serve as effective means of price discrimination? What would their potential effects on ordinary posted price sales be? What proportion of unsold capacity should sellers optimally offer in clearance sale venues? At what prices? Should a higher degree of buyers' risk aversion result in higher or lower predictability of near future prices? How would different levels of available capacity (holding demand fixed at a known level) affect optimal pricing policies?

This paper provides a simple framework that permits the investigation of these issues. The analysis provides insights into the determination of profit-maximizing pricing policies and the allocation of available capacity between “risk-free” and “risky” channels. We also attempt to explain why the use of multiple distribution channels may be more prevalent in technology-enabled markets.

Example

We begin the discussion by introducing a numerical example. A cruise line operator has 400 cabins left for sale on one of her ships. There are 1000 potential clients in the market whose values (v) for the cruise are uniformly and independently distributed between \$0 and \$1000. Suppose that buyers in this market are known to be highly risk-averse; once they set their minds on a specific cruise and commit to time away from work they abhor changing their plans. The seller accordingly estimates buyers’ utility to be represented by the function $u(v,p) = (v-p)^{\frac{1}{4}}$ whenever the net surplus from consumption of cruise vacations is non-negative $(v-p \geq 0)$. In case a buyer’s does not purchase a cruise his utility is assumed to be zero.

The cruise operator has three possible selling methods: a posted price, a multi-unit online auction and a random “last minute” sale event, held with probability of α. The seller may use any combination of the three in designing a selling scheme and we assume that all buyers are kept fully informed regarding her choice.

The seller may choose to sell units for a posted price of \$700, auction off 15 units with certainty and then offer any remaining unit in a clearance sale 14 days before departure, but only with probability of α=50% (so that with probability of 50% some capacity may remain unsold).

As the optimal solution, we prescribe the following[1]

- Set the posted price at $658 per cabin.
- Announce that a "last minute" sale will take place with a probability of 79%, in which units will be sold for $600 each.
- Do not auction off units at any time.

In the resulting equilibrium, 303 cabins are sold immediately and 97 cabins are reserved for the "last minute" clearance sale. Consequently, the seller's revenues are $245,720 or about 2.4% higher than what she could obtain by charging only a spot price of $600 while selling all 400 available units with certainty.

1.2 Electronic commerce perspectives

"Before [the Internet], people would try and book early to get a bargain. Now, they will wait until the last minute."

Andrew Shelton, senior manager of leisure marketing at British Airways[2]

[Figure 1 about here]

The Internet is facilitating a revolution in day-to-day commerce and has resulted in far-reaching changes in consumers' behavior. In the Travel and Entertainment industry, for example, online buyers are typically handling vast information while effectively performing functions that until recently belonged exclusively to professional travel agents. The number of people shopping online in this sophisticated fashion continues to grow at a rapid rate. Southwest Airways, for example, reports that

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[1] This solution was derived by using equations (45)-(47) in the appendix.

[2] Source: New Media Age. "Strategic Play – British Airways: Smooth Flight", September 2, 2004.

in fiscal year 2006 73 percent of its revenue was generated from bookings on southwest.com[3]. Ryanair, the largest low-cost carrier in Europe, has ceased operating staffed reservation calling centers altogether. Consequently, firms such as British Airways are facing tremendous pressures to modify their traditional business practices in order to better compete and take advantage of this new environment.

Still, in the market for leisure travel, as well as in others, the development of new online selling practices is apparently in its initial stage, and many related managerial issues remain open. In particular, it seems that firms are taking different approaches in responding to growing consumer demand for online "last minute" transactions. As an illustration of this, Figure 1 depicts pricing data sampled from three different airlines' websites for flights from London to New York. Similar to the pattern shown, we observe that on several occasions American Airlines dramatically dropped airfares on its flights about 72 hours before their departure but neither European competitor on the route responded by introducing similar discounts in any of its direct sales channels. Some sellers facilitate buyers' access to their "last minute" promotions by advertising them directly on their main websites, but comparable firms choose to keep a greater degree of separation between their regular and discount channels by partnering with third-party intermediaries[4] to offer tickets in auctions or clearance sales. In checking such sites, we often fail to find any hypertext *link* between the company's main website

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[3] Source: www.southwest.com.

[4] British Airways has partnered with LastMinute.com, which has recently become the largest online travel agency in Europe. Royal Caribbean has partnered with Atlantic International Travel in operating the website royalcarib.com, and it also makes extensive use of "last minute" specialists such as vacationstogo.com and auction websites such as skyauctions.com.

and any partners' discount websites[5]. Furthermore, in order to discourage later purchases, many sellers provide buyers with free insurance against future price reductions. Royal Caribbean, for example, has put in place just such a policy, applicable to all published fares within its direct sales channels. What could be the reasons for the different approaches taken by similar companies in managing their online "risky" channels? We will return to this issue later.

In view of the many practical implementation issues of price discrimination with random prices[6] we regard information technology as playing a central role for two reasons. First, sellers' decision rules in selecting a pricing policy often involve complex calculations and require accurate and timely information regarding demand and unsold capacity level. Second, for profitable segmentation to materialize buyers must be afforded easy access and timely information regarding more than one channel.

The Internet has made possible new and more elaborate selling methods that would be difficult to implement without computing power, such as multiple-unit auctions [see Pinker et al. (2003)]. Moreover, sellers can offer items on a number of web channels at the same time without incurring large incremental costs, such as inventory holding costs, for each featured channel. Novel technologies—online inventory and customer data management systems—give sellers a new ability to set their prices dynamically over time in order to maximize profits [see Choudhary et al. (2002). In

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[5] Interestingly, this type of seller behavior is observed even in cases where the same seller owns the discount website. For example, Dell.com is not linked to DellAuctions.com.

[6] A more precise definition would be "Price discrimination with random prices *or* availability". Throughout the paper, however, we model unavailability by the posting of an exceedingly high price. We explain this further when constructing our model in §2.

turn, buyers are presumably making some effort to learn the price patterns that are typical for the products that they are likely to purchase in the near future.

The Internet has also reduced buyers search costs significantly [see Bakos (1997)] and new questions arise regarding the impact that this may have on sellers' profitability and competitiveness. Clemons et al. (2002) argue that significant price dispersions exist in both commodity and differentiated product e-markets. They provide evidence that sellers who use the Internet do not necessarily compete on price alone and that many buyers are willing to pay a premium in order to purchase a good online from a seller of their choice. In this paper, we provide an argument that the reduction in buyers' search costs may *increase* sellers' segmentation benefits by allowing them to position items on multiple channels while facilitating buyers' choices as to the price or sales scheme that fits them best.

As a footnote to this discussion, we wish to stress that although electronic commerce considerations were a large part of our motivation, we do not regard the basic economic behavior studied in this work as being specific to the Internet. A car dealer who puts a strict time limit on her price offer, an auctioneer who uses a Dutch auction for the sale of flowers, or a department store that occasionally displays designer shoes down in its bargain basement are all practicing market segmentation with risk.

1.3 Related literature

The potential segmentation benefits that may arise from price randomization have long been recognized. Stiglitz (1982) suggests that incentive schemes yielding random outcomes may be desirable when agents are risk-averse but does not describe optimal policies that involve such randomization. In contrast, Riley and Zeckhauser (1983) show that a deterministic one-price scheme is optimal when buyers are risk-neutral and the seller has no capacity constraint. In two independent seminal studies, Matthews (1983) and Maskin and Riley (1984) characterize optimal auctions with risk-averse buyers under different sets of assumptions. While assuming, as we do in this paper, that buyers have uniform utility functions and differ only in their valuation of the good, both studies establish that the seller can devise a truth-revelation mechanism that strictly dominates any one-price scheme while inducing an equilibrium in which almost all buyers are faced with risk. With such an "optimal auction" every buyer is induced to reveal his value of the good; he is then assigned a schedule that includes a "bid submission" fee, a probability of winning the item, and an "acquisition price" to be paid only if the item is won. Matthews establishes that the acquisition price of any schedule should optimally be deterministic when buyers exhibit constant absolute risk aversion (CARA). Maskin and Riley deviate from this rather restrictive assumption at the cost of not obtaining necessary and sufficient conditions for the optimality of their suggested mechanism. Furthermore, it appears that the main barrier to the implementation of mechanisms resembling such "optimal auctions" in real-world markets is their inherent complexity. In fact, to the best of our knowledge no such selling scheme has ever been used.

Varian (1980) explains price dispersions in markets in which multiple sellers compete for sales of a homogenous good. He argues that when buyers differ in their ability to access price information the optimal selling scheme involves price randomization as the unique symmetric equilibrium outcome. The motivation behind such randomization is the desirability of avoiding head-on Bertrand competition. Baye and Morgan (2001) extend this model to include a monopolistic electronic intermediary that facilitates the transmission of price information while charging participating sellers and buyers nominal access fees. Interestingly, they find that at the resulting equilibrium, a seller's decision to participate in the electronic market takes the shape of a random event with probability α while its advertised price is a random variable drawn from a continuous distribution $F(p)$. Both papers are concerned with an oligopolistic environment. We, however, consider a monopoly seller, and our paper is therefore an alternative exploration of the problem of characterizing optimal price randomization schemes.

Price uncertainty can be thought of as being but one type of non-pecuniary cost that buyers may incur upon selecting a channel. Relevant references can therefore be found within a much wider range of optimal screening literature, which deals with the considerations of one or more sellers facing a heterogeneous population of buyers. Mussa and Rosen (1978) analyze the case in which all consumers agree on some ordinal ranking of product qualities but differ in their willingness to pay for higher-quality products. Several later research papers investigate optimal vertical differentiation policies to be used by a monopolist in such settings. Deneckere and McAfee (1996), for example, show that even when it is costly to *reduce* the quality of a good, a monopolist may actually choose to do so (or, in their language, "damage" the goods) in order to obtain screening benefits. Another strand of relevant literature (see, e.g., Gerstner and Holthausen (1986)) is concerned with cases in which price discrimination through channel is achieved by imposing inefficient time and effort participation requirements on buyers. An obvious example of that is the commonplace use of coupons and rebates in various product markets.

In what follows we analyze a simple two-period model of segmentation with random prices. We assume a specific utility function and only two possible available pricing schemes: a "risk-free" channel which consists of a single posted price, and a "risky" channel in which price is a random variable drawn from a probability distribution. We do not impose any restrictions on the shape of this distribution. Within this framework, we are able to fully characterize the profit-maximizing policy and investigate its behavior under different degrees of risk aversion among buyers and different levels of available capacity for the seller. We show that when the seller's available capacity is unlimited and buyers exhibit strict risk aversion, the optimal vehicle of price randomization is a discrete two-point distribution. In contrast, when buyers are risk-neutral *or* when the seller's available capacity falls below a threshold that is a concave function of the degree of buyers' relative risk aversion, the optimal policy is a one-price scheme.

One of our main findings is that a higher degree of risk aversion influences the monopolist to direct a higher proportion of available capacity to random price sales. At the same time, the seller reduces the effective risk involved in transacting through this channel (e.g., increases the frequency of sale events as well as the expected discounts). With increased risk aversion, buyers with the highest values are more reluctant to forgo a risk-free purchase and potentially compromise their exercisable surplus. Hence, the posted price is increased to reflect the higher "insurance premium" that those buyers are willing to pay. As a result, the market for posted price sales becomes smaller and a portion of available capacity is untied. In turn, this merchandise is targeted toward buyers with lower values through the use of the random price venue. Since the monopoly may gain nothing by inflicting further transaction risk on buyers with the lowest values, and at the same time a smaller degree of price uncertainty is sufficient to prevent leakage from the highest segments of the market, the risks embedded in the selling mechanism are mitigated as capacity allocation tilts in favor of random price sales.

The remainder of the paper is organized as follows. In §2 we construct the basic (uncapacitated) model and analyze the resulting equilibrium (Theorems 1 and 2). Section 3 includes three extensions to the basic model. In the first, the monopolist has less than full control over the design of the selling mechanism. In the second, an arbitrary number of risk-neutral re-sellers compete with the monopolist in the retail market. The third extension entails the addition of a capacity constraint to the model (resulting in Theorems 3 and 4). In §4, we discuss some welfare implications of both uncapacitated and capacitated models. In §5 we derive the managerial implications of the model and offer some additional concluding remarks.

2. Model

We consider the problem of a risk-neutral seller who wishes to maximize her expected revenue from the sale of multiple units of a good. The seller is a monopolist and we normalize her constant marginal cost of production to be zero. For this part of the analysis, we also assume that the seller's production capacity is unlimited.

Suppose the seller initially offers the good at a price p_1. If potential demand and production capacity are not totally exhausted, it pays for the seller to continue and then offer additional units at a lower price p_2, targeting potential buyers that have so far chosen not to buy at p_1. Since buyers understand this, they anticipate the subsequent discount, and optimally wait for it. In the absence of a time deadline, similar considerations apply indefinitely. The seller's power to attain strictly positive profits (inducing potential customers to buy at a non-infinitesimal positive price) depends on the possibility of a **commitment,** whereby some potentially profitable future decisions are inhibited. The ex-post opportunity loss (which appears to violate subgame perfection) is amply justified by the ex-ante profitability of the initial commitment.

With any non-degenerate pure strategy, all sales take place at the same time period. Recognizing the possibility (indeed inevitability) of commitment, we consider an alternative "two-period" selling strategy that involves randomization. In the first period, the seller offers the good for sale at some posted per unit price p_1; in the second period, the price p_2 is a random draw from a discrete[7] probability distribution $f(X, A)$. The term $X = \{x_i\}_{i=1}^{n}$ is a vector of n non-negative prices that are indexed in an increasing order ($i < j \rightarrow x_i \leq x_j$), and $A = \{a_i\}_{i=1}^{n}$ is a vector of n probabilities such that $\Pr[p_2 = x_i] = \alpha_i$ for all $\{i = 1,2,..,n\}$, and $\sum_{i=1}^{n} \alpha_i \leq 1$.

Using the above terms, we let the triplet $S = \{p_1, X, A\}$ represent the seller's "pricing policy". The seller announces the policy S at the beginning of the first period and we assume that she can indeed credibly commit to truthfully following it.

The market comprises a large number of buyers whose values for the good are independently and uniformly distributed over an interval of a unit measure $v_i \sim U[0,1]$. Each buyer's demand is for a single unit of the good. Buyers are risk-averse and their preferences are uniformly represented by the utility function $u(v_i, p) = (v_i, p)^{1-\rho}$. The parameter ρ is common to all buyers and may range between 0 and 1; in related literature, this term is often referred to as the *degree of buyers' relative risk aversion*[8].

At the beginning of the first period, all buyers freely observe the policy S. Each buyer then individually chooses whether to purchase the good at a price of p_1 or delay his decision until the next period. In the second period, the realization of the random variable p_2 becomes known and all buyers

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[7] Although we consider only discrete price distributions, any interesting continuous distribution can be approximated by a discrete form, so no generality is lost.

[8] For a utility function *u(y)* the degree of relative risk aversion is defined by $r_R = -yu''(y)/u'(y)$.

whose values exceed this price make their purchases. At the same time, buyers with values lower than p_2 end up with no purchase and with a utility of zero.

2.1 Buyers' behavior

A buyer whose value is v will optimally purchase the good in the first period if and only if the following two conditions are satisfied

$$v \geq p_1. \tag{1}$$

$$\Delta(v) = (v - p_1)^{1-\rho} - \sum_{i=1}^{m(v)} \alpha_i (v - x_i)^{1-\rho} \geq 0. \tag{2}$$

Where $m(v)$ is the index of the highest price not exceeding v. The function $\Delta(v)$ represents a buyer's excess utility from a risk-free purchase. It can be shown that $\Delta(v)$ is everywhere continuous with respect to buyers' value but is not always monotone with respect to it. The following lemma offers an important necessary condition for "risky" channel participation.

Lemma 1: *A buyer will delay his purchase in the first period only if he expects an (average) price discount to materialize in the second:* $\Delta(v) \leq 0 \rightarrow p_1 \geq \sum_{i=1}^{n} \alpha_i x_i$*, all* $v \in [p_1, 1]$.

All proofs in this paper are contained in the appendix. Lemma 1 seems intuitive. We learn from it that even though positive probabilities may be assigned to second period prices that are higher than the first period posted price (p_1), the average revenue generated to the seller by any second period buyer would always be lower than that of a first period buyer.

In the next sub-section, we study the family of policies that include a two-point price distribution in the second stage ($n = 2$). Narrowing this subset even further, we restrict one of those prices to be sufficiently high that no buyer will be willing to pay it. This form is of special importance, as we show at the next section that the seller can always find an optimal policy that belongs to it.

2.2 The "two-price" policy form

Consider a situation in which the seller posts in the first period a “spot” price of p_1 and announces at the same time that with probability α she will hold a random “sale” in the second period, and that the “sale” price will be p_2. If she does not hold a “sale” the seller will not sell the product in the second period at all.

This takes the form of the following policy

$$S = \{p_1, X, A\}\,, \qquad X = \{p_2, h\}\,, \qquad A = \{\alpha, 1-\alpha\}\,, \qquad p_1 \geq p_2\,, \qquad 0 \leq \alpha \leq 1.$$

where h is any number (weakly) greater than 1, which is the highest possible value a buyer can assume in our model. Throughout the paper we shall use the convention $h = 1$. In this particular case, it can be shown that the buyers’ excess utility from risk-free purchase $\Delta(v)$ is a monotonically increasing function with respect to v for any parameter value and therefore has, at most, a single crossing at a level of zero. This threshold value can therefore assume a functional form

$$b = B(p_1, p_2, \alpha, \rho) = \min\left(\frac{p_1 - \alpha^{\frac{1}{1-\rho}} p_2}{1 - \alpha^{\frac{1}{1-\rho}}}, 1\right). \tag{3}$$

All buyers with values higher than b optimally make purchases in the first period and all buyers with values between p_2 and b make purchases in the second period whenever a “sale” is held. Quite expectedly, the function B is monotonically increasing with respect to both p_1 and α and monotonically decreasing with respect to both p_2 and ρ.

2.3 The seller’s behavior and the characterization of a profit-maximizing policy

The seller’s aim is to select a policy in order to maximize her expected profits when all buyers are kept fully informed and behave rationally. Without any loss of generality, we restrict our attention to the domain $\Theta(\rho)$ of policies that induce a partition of buyers into first and second demand sets that are contiguous and touching at a single point $b = B(S;\rho)$. Hence, if a buyer prefers to make his purchase in the first period, then so will all buyers with values higher than his.[9]

To save on notations we use simply Θ and b. The seller's optimization problem takes the form

$$\max_{S\in\Theta} p_1(1-b) + \sum_{i=1}^{m(b)} \alpha_i(b-x_i)x_i\,. \tag{4}$$

Before proceeding to the solution, we provide a simple example in order to illustrate the considerations involved in the seller's selection of a policy. For this example, suppose that the seller compares only the following two policy alternatives:

$$S_1 = \left\{\frac{1}{2}, \{0\}, \{0\}\right\}.$$

$$S_2 = \left\{\frac{2}{3}, \left\{\frac{1}{3}, 1\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}\right\}.$$

We shall refer to S_1 as "the benchmark policy'". This is the optimal one-price policy by which the seller may turn a profit of $\frac{1}{4}$ simply by charging a fixed price of $\frac{1}{2}$ in a single period. The first period demand resulting from policy S_2 is given by $(1-b)$, where b is described by equation (3); the second period demand is accordingly $\frac{1}{2}(b-\frac{1}{3})$ and the overall profit from this strategy is given by

$$\Pi(S_2) = \frac{5\cdot 2^{\frac{1}{1-\rho}} - 8}{18\cdot\left(2^{\frac{1}{1-\rho}} - 1\right)}.$$

The profit function above is monotonically increasing in the degree of buyers' risk aversion and exceeds the benchmark profit if and only if ρ is greater than 0.644. Figure 2 assists us in describing

[9] As mentioned in the previous section, this type of equilibrium is induced by all two-price policies.

the different effects that come into play in the comparison between the alternative equilibria under policies S_1 and S_2. In segment I, using policy S_2 provides better surplus extraction. At the same time, the seller loses revenues from buyers whose values fall within segment II due to cannibalization (or *leakage*). In segment III the seller benefits from extending her market to include new buyers with lower values. The key challenge is identifying a policy that yields an overall increase in profits as the sum result of all three effects.

[Figure 2 about here]

Our first Theorem asserts that when buyers are risk-neutral the seller cannot achieve a profitable segmentation.

Theorem 1: *Let buyers be uniformly risk-neutral ($\rho = 0$) and let $\hat{S} = (\hat{p}_1, \hat{x}, \hat{\alpha})$ be a policy that induces a non-empty subset of buyers to make purchases in the first period. Then,*

$\Pi(\hat{S}) \leq \hat{p}_1(1 - \hat{p}_1)$.

Surprisingly, we find that the fact that introducing a random second period price is never optimal does not depend on the optimality of the first period price in a policy. Our next task is to show how buyers' risk aversion may change the nature of the seller's solution. Intuitively, a sufficiently high degree of risk aversion may mitigate the negative effects of cannibalization to such an extent that the seller can exploit price uncertainty in order to increase her profit. In fact, we find that for *any* degree of risk aversion the seller can indeed find an optimal two-price policy that results in a higher profit than the fixed-price benchmark. We now formalize our main result.

Theorem 2: *For any degree of strict risk aversion ($0 < \rho < 1$) there exists a profit-maximizing policy $S^*(\rho) = \{p_1^*(\rho), x^*(\rho), \alpha^*(\rho)\}$ that involves two distinct second period prices $x^*(\rho) = \{p_2(\rho), 1\}$ to be charged with two corresponding strictly positive probabilities $\alpha^*(\rho) = \{\alpha(\rho), 1 - \alpha(\rho)\}$. This optimal policy strictly dominates any one-price scheme ($\Pi(S^*) > \frac{1}{4}$).*

A central implication of Theorem 2 is that the optimal vehicle of price randomization within a two-channel structure is a draw from a discrete two-point probability distribution function where one of the points represents a price that is prohibitively high.[10]

2.4 Comparative Statics

In this section we describe the effects of changes in the degree of buyers' risk aversion on various equilibrium variables of the model. Equations (5) and (6)[11] formulate the two prices of the profit-maximizing policy:

$$p_1^* = \frac{2\left(1-(\alpha^*)^{\frac{1}{1-\rho}}\right)}{4-\alpha^*\left(1+(\alpha^*)^{\frac{\rho}{1-\rho}}\right)^2}. \tag{5}$$

$$p_2^* = \frac{\left(1+(\alpha^*)^{\frac{\rho}{1-\rho}}\right)\left(1-(\alpha^*)^{\frac{1}{1-\rho}}\right)}{4-\alpha^*\left(1+(\alpha^*)^{\frac{\rho}{1-\rho}}\right)^2}. \tag{6}$$

where the probability α^* is determined as the unique feasible solution to the following equation

$$\left(\alpha^{\frac{1+\rho}{1-\rho}}+\alpha^{\frac{1}{1-\rho}}\right)\left(\frac{\rho}{1-\rho}\right)-\alpha^{\frac{\rho}{1-\rho}}\left(\frac{1+\rho}{1-\rho}\right)+1=0. \tag{7}$$

We use the above derivations to investigate the relationship between the buyers' risk aversion and the seller's optimal policy variables (see Figures 3A and 3B).

[Figure 3 about here]

It is an intuitive result that the first period price is strictly increasing with risk aversion (see Figure 3A); the buyers' increased reluctance to incur price uncertainty intensifies the demand for purchases

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[10] We have not been able to find a strong economic intuition behind this result.

[11] These are derived in the appendix as part of the proof of Theorem 2,

in the first period and results in a correspondingly higher posted price (p_1^*). At the same time, the "sale" price (p_2^*) of the second period is a decreasing function of buyers' risk aversion since a reduced cannibalization effect allows the seller to charge a price that is closer to the *ex post* efficient level given by $\frac{B(S^*;\rho)}{2}$ (this is the price that the seller would charge in the second period if she were to defect from truthfully following her declared policy). Interestingly, the optimal probability of a "sale" (α^*) is monotonically increasing with risk aversion (see Figure 3B). In other words, in environments where buyers would be more reluctant to incur transaction risk the model advocates installing mechanisms that assign higher probabilities to lower prices. Still, as Figure 3C shows, with increased risk the ratio of average sales in the two periods always tilts in favor of the second period channel (!). We later generalize this result in our Theorem 4 to include also a binding capacity constraint. Finally, it is an obvious result that the monopoly's payoff increases monotonically with risk aversion (see Figure 3D). This phenomenon is a direct consequence of the monopoly's enhanced capacity to segment its market.

3. Model Extensions

3.1 Optimal policies with exogenous risk

Theorem 2 asserts that a monopolist can always find a profitable segmentation scheme in a market comprising (strictly) risk-averse buyers. In this section, we argue that this fundamental result can be generalized to include a case in which the probability α is not optimally set (we refer to this as "exogenous risk"). With this model, we attempt to get a flavor of optimal behavior when a seller considers using a sub-optimal mechanism for some reason, as when a third-party retailer controls the probability of a "sale" but the wholesaler still maintains control over prices and supply quantities.

In the following corollary we use the framework developed so far in order to argue that for any exogenously given $0 < \alpha < 1$ there exists a pair of prices p_1 and p_2 such that the resulting two-price policy strictly dominates any one-price scheme.

Corollary (Theorem 2): *Let α and ρ both be arbitrary parameters between 0 and 1. Then, the policy $S^*(\rho) = \{p_1^*(\rho,\alpha), \{p_2^*(\rho,\alpha), 1\}, \{\alpha, 1-\alpha\}\}$ with $p_1^*(\rho,\alpha)$ and $p_2^*(\rho,\alpha)$ given by equations (5) and (6) respectively, is optimal.*

This result stems directly from the proof of Theorem 2. We follow with a short discussion of related comparative statics. Figure 4 may be regarded as an abstract view of the optimal design of a dual channel that comprises a spot and a random price. In that sense, scenarios in which α is closer to zero may better represent mechanisms that are "riskier" from the buyers' standpoint. We find that the difference between the two channel prices is a decreasing function of the parameter α. The "sale" price (p_2) is monotonically increasing, but the shape of the optimal posted price (p_1) as a function of α is unimodal, and its mode α^* corresponds to the point of maximal profits (note that the optimum value in Theorem 2 can be expressed as $\Pi(S^*) = p_1^*/2$). When α is high (and known to buyers), the perceived risk of delaying purchases to the second period is correspondingly low, so the cannibalization effect renders large deviations from period prices of $\frac{1}{2}$ unprofitable. When α is very low (i.e., the mechanism is "highly risky") the seller may capture only a small fraction of the surplus of buyers who are still willing to defer their purchases. The impact of the second period sale on total revenues becomes marginal and segmentation benefits do not justify large deviations from a first period price of $\frac{1}{2}$. At the point α^*, which is always in the interior, the tradeoff between cannibalization and extraction efficiency results in the maximum possible profit.

[Figure 4 about here]

Next, we investigate the optimal allocation of capacity between spot and random price channels. Figure 5 shows total average sales and their corresponding decompositions into each of the two channels. Interestingly, we learn that optimal seller behavior entails assigning a higher proportion of the total sales capacity to the second period sales channel whenever the transaction risk involved in it is lower. From the channel designer's point of view, it should be noted that analyzing the performance of each channel separately within the two-channel structure might be misleading. As is apparent from the figure, the fact that the second period revenues are monotonically increasing with respect to α does *not* suggest that any transaction risk present in a distribution channel should be mitigated or eliminated in all circumstances.

[Figure 5 about here]

3.2 Third-party re-sellers

In the preceding analysis we assumed that the seller is not only the monopoly producer of the good but also the only risk-neutral agent in the market. We now consider an alternative case in which an arbitrary number of risk-neutral re-sellers also exist in the market. The re-sellers are not authorized by the wholesaler and are therefore compelled to buy the good at a forward retail price in the hope of selling it later at a profit once it appreciates. We assume that no re-seller derives any direct utility from consumption of the good. We also ignore some implications, such as lack of warranty, and assume that all buyers are indifferent between buying from the monopoly producer and buying from a re-seller. Would it be possible for any number of such re-sellers to gain arbitrage profits in the market? The answer to this question is negative, as we show.

In a realization in which the producer does not hold a "sale" let us assume that all re-sellers charge the price ψ in a symmetric equilibrium. Each re-seller's expected (per unit) profit is then given by

$$\Pi_{rs}(\psi) = (1-\alpha)\psi + \alpha p_2 - p_1. \tag{8}$$

The minimum equilibrium price ψ that would result in a non-negative profit for a re-seller Therefore is

$$\psi_{min} = \frac{p_1 - \alpha p_2}{1-\alpha}. \tag{9}$$

However, since the utility function of buyers is concave, it is straightforward to verify that no buyer will be willing to buy at this price. Indeed, by the definition of strict concavity we get

$$\alpha(v-p_2)^{1-\rho} + (1-\alpha)(v-\psi_{min})^{1-\rho} < (v-p_1)^{1-\rho}. \tag{10}$$

We conclude that the monopoly's capacity to segment the market with risk profitably may not be impaired by the presence of risk-neutral re-sellers.

3.3 Limited capacity

We now incorporate a capacity constraint into the model and analyze the resulting equilibrium. We show that when merchandise is in short supply relative to demand, the seller optimally should charge only one price.

We assume that buyers freely observe the seller's available production capacity (denoted k) in the first period. The entire lot is readily available for sale in the first period, and the seller can costlessly carry over any unsold unit to the second period. We do not model "stock outs"; that is, we assume that within any chosen policy the seller does not assign positive probabilities to scenarios in which demand exceeds supply. Hence, the minimum price that may be charged at any equilibrium is $(1-k)$. We obtain the following result:

Theorem 3: *Let ρ be the degree of buyers' relative risk aversion ($0<\rho<1$) and let k be the seller's overall (two period) available capacity ($0 \le k \le 1$). There exists a profit-maximizing policy $S^*(\rho,k) = \{p_1^*(\rho,k), x^*(\rho,k), a^*(\rho,k)\}$ that involves two second period prices $x^*(\rho,k) = \{p_2^*(\rho,k), 1\}$ and two corresponding probabilities $a^*(\rho,k) = \{\alpha(\rho,k), 1-\alpha(\rho,k)\}$ and strictly* *dominates any one-price charge, if and only if buyers are risk-averse ($\rho > 0$) and the seller's capacity exceeds a concave threshold function: $k > \frac{1-\rho}{2-\rho}$.*

Figure 6 depicts the two regions of Theorem 3. When production capacity is limited the seller targets buyers with higher values for the good and has little or no incentive to use segmentation tools in order to include buyers with relatively low values. Both effects result in optimal channeling of a smaller proportion of available capacity to the second period, or even the elimination of second period sales when capacity is low.

[Figure 6 about here]

The optimal policy under capacity constraints is described in the proof of Theorem 3, in the appendix. We use this solution to establish the following result

Theorem 4: *For any given level of available capacity (k), an increased degree of buyers' risk aversion will result in optimal allocation of a higher proportion of k to second period ("risky channel") sales.*

Theorem 4 is somewhat counter-intuitive. We often observe in consumer markets that a seller reacts to buyers' shifting preferences by modifying the product in the same direction as this change in preferences. In Theorem 4, however, we establish the opposite; when buyers are generally more reluctant to incur transaction risk the seller optimally involves such risk in the sale of more units.

4. Welfare implications

We define total welfare as the aggregation of net expected consumer surplus and the seller's equilibrium profit: $W = CS + \pi$. Under the optimal two-price policy of both uncapacitated and capacitated cases, we calculate the equilibrium values of total welfare as the sum of the following two arguments

$$CS = \frac{(1-p_1)^2 - (1-\alpha)(b-p_2)^2}{2} \; and$$

$$\pi = (1-p_1)p_1 + \alpha(b-p_2)p_2.$$

Since we assume zero marginal costs of production, whether equilibrium social welfare increases or decreases when compared to a one-price monopoly benchmark depends entirely on the level of production. When available capacity is unlimited, the total social welfare always increases as a result of increased production. When the capacity constraint is binding in equilibrium, however, social welfare may decrease. A less intuitive result pertains to the welfare of consumers as a whole. We find that regardless of risk aversion and available capacity levels consumers are always worse off in the aggregate than they are in the one-price monopoly scenario. This result holds in an even stronger sense because the above formulation of consumers' surplus does not incorporate buyers' disutility from bearing risk. Notably, an incremental increase in the population's risk aversion has opposite effects on different buyer groups. Buyers with the highest values, who would typically commit to a risk-free purchase, are hurt as a result of the consequent increase in first period price, but buyers with lower values, who would typically prefer to delay their purchase to the second period, are better off because increased risk aversion entails a lower second period "sale" price or a higher frequency of sales at the new equilibrium.

5. Concluding remarks

Our primary conclusion is that firms with monopoly power can increase their profits by using price randomization whenever buyers are risk-averse and available sales capacity is not in severe shortage relative to expected demand. In cases when such a potential existed, we described optimal pricing and capacity allocation between "risk-free" and "risky" channels to be taken by a seller in exploiting it. The characterization of such profit-maximizing policies proved a fairly complicated matter even within the simplified framework used; more so, it undoubtedly constitutes a significant challenge in real market settings. The dynamic nature and rapid growth of new electronic markets underscore the importance of a better understanding of related issues and warrant the development of further theoretical foundations.

Two managerial implications of the model are worth emphasizing. The first one is that a monopoly seller should mitigate the extent of risk involved in the selling mechanism in response to increased buyers' risk aversion. Within the optimal two-price policy form resulting from the analysis, such mitigation translates to an increase in the frequency of "last minute" sales. In most product markets, however, controlling for buyers' risk by altering the frequency of sales may lack immediate effectiveness since an extended time may elapse before most buyers learn of the change and internalize it in their purchasing behavior. In settings with incomplete information, an alternative practical approach for sales-channel risk management can also be found in the dissemination of different amounts of information by the seller (extant literature relates to this area see, e.g., Milgrom & Weber (1982)). For example, priceline.com often reveals prior winning bids to potential participants in its auctions.

A second practical implication of the model is that when risk aversion is held at any constant level, and the frequency of clearance sales is *exogenously* determined, the seller should optimally channel higher volume through the second period ("risky") channel whenever this frequency is higher (see Figure 5). Simply put, an optimal policy entails the equilibrium allocation of fewer transactions to a "riskier" channel within a multi-channel selling scheme. Interestingly, this effect may result in an agency problem when an outside retailer is contracted to carry out "last minute" transactions. Although a global (two-channel) profit-maximizing policy often involves some non-trivial desirable extent of risk to be installed in the selling mechanism, a contracted retailer will *unambiguously* have an incentive to unilaterally mitigate this risk thus augmenting her channel's revenue share and resulting commissions. Since the potential benefits from such deviations are more substantial when the degree of optimal mechanism riskiness is relatively high (corresponding to relatively low degrees of buyers' risk aversion), we expect such outsourcing to be more prevalent in markets comprising highly risk-averse buyers.

In our opening remarks we observed that Carnival Cruise Lines facilitated buyers' participation in "last minute" transactions, but a comparable firm, Royal Caribbean, seemed to have actively discouraged such speculative behavior. We can use the framework developed here to discuss possible reasons for this. Our model predicts that the effect of both a higher degree of buyers' risk aversion and a greater extent of excess capacity will result in an increased segmentation potential from using a two-channel scheme, and can thus render the use of "risky" sales more attractive for a seller. In this case, however, we doubt that substantial differences in risk aversion existed between the populations of buyers, as both cruise lines operated in same regional markets and offered comparable product mixes. Unfortunately, with respect to observed differences in demands and capacity for the two cruise lines, we find that the predictions of the model *conflict* with the actual sellers' behavior. Pertinent financial data reported[12] clearly indicate that Carnival experienced a stronger demand for cruise tickets than Royal Caribbean did yet Carnival apparently used the "risky" channel more extensively.

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[12] We extracted financial data from the Carnival Corporation and Royal Caribbean Cruises Ltd. 2003 annual reports.

Capacity utilization rates were 103.4% for Carnival and 103.2% for Royal Caribbean. Passenger capacity is calculated based on two passengers per cabin. Revenues from passenger tickets per Available Lower Deck Berth Day (ALBD). The figures were $151.27 and $142.69 per ALBD for Carnival and Royal Caribbean, respectively. Product quality or brand perception differences cannot, in our opinion, explain the yield differences in this case.

Our model indicates that Royal Caribbean would likely have improved its profitability by emulating Carnival's more direct "last minute" selling approach. A direct comparison of key financial performance measures[13] pertaining to the efficacy of overall yield management for the two companies lends this further support. It should be noted, however, that considerations that were left outside the scope of our discussion might give rise to opposing arguments. For example, it is possible that Royal Caribbean's management aimed to achieve a completely different sort of price discrimination by targeting only *well-informed* buyers for the marketing of special promotions (for analyses of this type of seller's behavior see, e.g., Salop and Stiglitz (1977), Varian (1980), Baye and Morgan (2001)). Further research is required in order to determine more clearly under what circumstances either type of segmentation should be preferred.

Another limitation of our model is that we consider a monopoly setting and thus ignore the impact of strategic interactions in a competitive environment. It is quite possible that competition will dramatically reduce the benefits of segmentation, as deliberately imposing transaction risk on buyers may lead to an unsustainable equilibrium outcome. In markets that possess a relatively low degree of horizontal differentiation, we expect that buyers will tend to purchase a close substitute when their most preferred product is unavailable or its price is deemed too high. Incorporation of different transaction costs for buyers or different venue costs for the seller, nonlinear production cost functions, endogenous product qualities, and stochastic demand and supply could also contribute significantly to this discussion. We leave those interesting issues as topics for future research.

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[13] The Carnival Corporation achieved a 4.94% rate of return on its assets (pro-forma data) and an 8.77% return on shareholders' equity (on a diluted basis); the comparable Royal Caribbean Cruises Ltd. figures were 2.47% and 6.58%, respectively.

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Appendix: Proofs

<u>Lemma 1:</u> The utility function is concave and monotonically increasing with respect to value. Hence, $\Delta(v) < 0$ implies

$$0 > (v - p_1)^{1-\rho} - \sum_{i=1}^{m(v)} \alpha_i (v - x_i)^{1-\rho} \geq (v - p_1)^{1-\rho} - \left(\sum_{i=1}^{m(v)} \alpha_i (v - x_i) \right)^{1-\rho}$$

$$\geq (v - p_1)^{1-\rho} - \left(v - \sum_{i=1}^{m(v)} \alpha_i x_i \right)^{1-\rho} . \tag{11}$$

Therefore, $p_1 > \sum_{i=1}^{m(v)} \alpha_i x_i$. □

Theorem 1: The set of equilibrium first period buyers is non-empty. If the set of equilibrium second period buyers is empty then the assertion of the theorem is trivially satisfied. Let us assume, then, that the set of second-period buyers also is non-empty. The function $\Delta(v)$ given in equation (2) is continuous and its domain a closed interval; under the risk-neutrality assumption it is also monotonically increasing. Hence, by the mean value theorem there exists a *unique* value $b \in (0,1)$ such that $\Delta(b) = 0$. The seller's profit function is

$$\Pi(\hat{S}) = \hat{p}_1(1 - b) + \sum_{i=1}^{m(b)} \hat{\alpha}_i (b - \hat{x}_i) \hat{x}_i. \tag{12}$$

By the incentive constraint of the indifferent buyer (whose value is b), we obtain

$$\hat{p}_1 = \left(1 - \sum_{i=1}^{m(b)} \hat{\alpha}_i \right) b + \sum_{i=1}^{m(b)} \hat{\alpha}_i \, \hat{x}_i. \tag{13}$$

For notational convenience we use the cumulative distribution form $F(b) \equiv \sum_{i=1}^{m(b)} \hat{\alpha}_i$. With this term, we describe the following revenue equivalent policy $\bar{S}$:

$$\bar{S} = \{\hat{p}_1, \bar{X}, \bar{A}\}; \quad \bar{X} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_{m(b)+1}\} = \{\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{m(b)}, b\},$$

$$\bar{A} = \{\bar{\alpha}_1, \bar{\alpha}_2, \dots, \bar{\alpha}_{m(b)+1}\} = \{\hat{\alpha}_1, \hat{\alpha}_2, \dots, \hat{\alpha}_{m(b)}, 1 - F(b)\}.$$

It is easy to verify that $\Pi(\bar{S}) = \Pi(\hat{S})$.

We show that the variance of the second period price, within the equilibrium induced by the optimal policy, must be zero. Note that by equation (13) the first period price equals the second period mean. The required variance term is formulated as follows:

$$\sigma^2 \equiv E[(\bar{x}_i - \hat{p}_1)^2] = \big(1 - F(b)\big)(b - \hat{p}_1)^2 + \sum_{i=1}^{m(b)} \bar{\alpha}_i (\bar{x}_i - \hat{p}_1)^2$$
$$= \big(1 - F(b)\big)b^2 - p_1^2 + \sum_{i=1}^{m(b)} \bar{\alpha}_i (\bar{x}_i)^2. \tag{14}$$

Equations (13) and (14) yield the following expressions, respectively:

$$\sum_{i=1}^{m(b)} \bar{\alpha}_i \, \bar{x}_i = \hat{p}_1 - \big(1 - F(b)\big)b. \tag{15}$$

$$\sum_{i=1}^{m(b)} \bar{\alpha}_i (\bar{x}_i)^2 = \sigma^2 + \hat{p}_1^2 - \big(1 - F(b)\big)b^2. \tag{16}$$

Once we incorporate the expressions for the LHS terms in (15) and (16) into the profit function (12), we get our required result:

$$\Pi\big(\hat{S}\big) = (1 - \hat{p}_1)\hat{p}_1 - \sigma^2. \tag{17}$$

From the non-negativity of the above variance term we know that the policy $\hat{S}$ is dominated by a policy where only one price $\hat{p}_1$ is charged, and our proof is thus complete. Note that we proved more than the Theorem asserted: not only does the seller's optimal policy consist of a single fixed price, but the profit function also is monotonically decreasing in the measure of second-period price variability (σ^2).□

<u>Theorem 2:</u> We first show that for any $\rho > 0$ there exists a two-price policy S such that $\Pi(S) > \frac{1}{4}$ and then argue that there does not exist an n-price policy that strictly dominates it. Let

$S = \{p_1, X, A\}$ with $X = \{p_2, 1\}$ and $A = \{\alpha, 1-\alpha\}$ represent an arbitrary two-price policy. Suppose that there exists a buyer in the market who is indifferent between a first and a second period purchase. Then, as we argued in §2.1, his value is given by

$$b = B(S) = \frac{p_1 - \alpha^{\frac{1}{1-\rho}} p_2}{1 - \alpha^{\frac{1}{1-\rho}}}. \tag{18}$$

Next, we incorporate this term into the profit function given in equation (4). For feasibility, we impose the restriction that b must lie between p_1 and 1 (constraint (19.2) forth). We have in hand the formulation of the seller's problem:

$$\max_{p_1, p_2, \alpha} p_1 \left(1 - \frac{p_1 - \alpha^{\frac{1}{1-\rho}} p_2}{1 - \alpha^{\frac{1}{1-\rho}}}\right) + \alpha p_2 \left(\frac{p_1 - \alpha^{\frac{1}{1-\rho}} p_2}{1 - \alpha^{\frac{1}{1-\rho}}} - p_2\right). \tag{19}$$

s.t.:

$$p_1 - p_2 \geq 0. \tag{19.1}$$

$$p_1 + \alpha^{\frac{1}{1-\rho}}(1 - p_2) - 1 \leq 0. \tag{19.2}$$

$$p_1, p_2, \alpha \in [0,1]. \tag{19.3}$$

Fortunately, we find that whenever $\rho > 0$ none of the above-listed constraints bind at any optimum so we ignore them in what follows.

First-order necessary conditions:

$$\frac{\partial \Pi(\cdot)}{\partial p_1} = \frac{1 - 2p_1 + p_2\left(\alpha + \alpha^{\frac{1}{1-\rho}}\right) - \alpha^{\frac{1}{1-\rho}}}{1 - \alpha^{\frac{1}{1-\rho}}} = 0. \tag{20}$$

$$\frac{\partial \Pi(\cdot)}{\partial p_2} = \frac{p_1\left(\alpha + \alpha^{\frac{1}{1-\rho}}\right) - 2p_2\alpha}{1 - \alpha^{\frac{1}{1-\rho}}} = 0. \tag{21}$$

$$\frac{\partial \Pi(\cdot)}{\partial \alpha} = \frac{(p_1 - p_2)\left(p_1 \alpha^{\frac{1}{1-\rho}} - p_2\left(\alpha(1-\rho) + \rho \alpha^{\frac{2-\rho}{1-\rho}}\right)\right)}{\alpha\left(1 - \alpha^{\frac{1}{1-\rho}}\right)^2 (1-\rho)} = 0. \tag{22}$$

The case when $\alpha = 1$ is of no significance in the model, so we can ignore the first terms on the right-hand sides of equations (20), (21), and (22). By combining equations (20) and (21), we get

$$p_1^* = \frac{2\left(1 - \alpha^{\frac{1}{1-\rho}}\right)}{4 - \alpha\left(1 + \alpha^{\frac{\rho}{1-\rho}}\right)^2} \quad and \tag{23}$$

$$p_2^* = \frac{\left(1 - \alpha^{\frac{1}{1-\rho}}\right)\left(1 + \alpha^{\frac{\rho}{1-\rho}}\right)}{4 - \alpha\left(1 + \alpha^{\frac{\rho}{1-\rho}}\right)^2}. \tag{24}$$

From equations (22),(23) and (24) we derive the following condition for α:

$$\left(\alpha^{\frac{1}{1-\rho}} + \alpha^{\frac{1+\rho}{1-\rho}}\right)\left(\frac{\rho}{1-\rho}\right) - \alpha^{\frac{\rho}{1-\rho}}\left(\frac{1+\rho}{1-\rho}\right) + 1 = 0. \tag{25}$$

Equation (25) does not have a closed-form solution so we use numerical methods to solve for α^*. Note that by equations (23) and (24) the entire optimal policy is uniquely determined by this value. Still we must verify that the numerical solution is indeed a global maximizer. We obtain the representation of profits under the hypothetical case in which α is treated as a parameter:

$$\Pi^*(\alpha) = \frac{\left(1 - \alpha^{\frac{1}{1-\rho}}\right)}{4 - \alpha\left(1 + \alpha^{\frac{\rho}{1-\rho}}\right)^2}. \tag{26}$$

This is a continuous and strictly concave function. Hence, we find the extreme point to be a global solution to the seller's problem, as required. Since it is true that $\lim_{\alpha \to 0} \Pi^*(\alpha) = \lim_{\alpha \to 1} \Pi^*(\alpha) = \frac{1}{4}$, the mode of maximum profit is always found at an interior point of the support ($0 < \alpha^* < 1$). The reader may refer to Figure 5 in the paper's text for an illustration of this. The Theorem's assertion that $\Pi^*(\alpha) > \frac{1}{4}$ for any $\rho > 0$ is thus proven.

It remains to be verified that there does *not* exist another policy S_n that includes an n-point second period price distribution ($n \geq 3$) and yields the seller a (strictly) higher payoff than the optimal two-point distribution we analyzed above does. Let:

$$S_n = \{\,{}_np_1,\ {}_nX,\ {}_nA\};\quad {}_nX = \{\,{}_nx_i\}_{i=1}^{n}\,,\ {}_nA = \{\,{}_n\alpha_i\}_{i=1}^{n}\,, n \geq 3.$$

and assume the contrary—that is, S_n is the profit-maximizing policy. Policy S_n with $n \geq 3$ may result in more than one indifference point b. We pick $\underline{b} = \underline{B}(S_n) = \min_v(v|\Delta(v) = 0)$ and by Lemma 1 get the following upper bound for equilibrium profits corresponding to S_n:

$$\overline{\Pi}(S_n) = (1 - \underline{b})p_1 + \sum_{i=1}^{m(b)} \alpha_i(\underline{b} - x_i)x_i. \tag{27}$$

Let us suppose for a moment that the seller, while making her policy decision, optimistically assumes that her payoff will be $\overline{\Pi}(S_n)$. Since S_n is supposed to maximize the seller's payoff, it must in particular dominate any policy S such that $p_1 \in S, p_1 = \ {}_np_1\ {}_{p_1 \in S}$, and $\underline{b} = \underline{B}(S) = \underline{B}(S_n)$. We therefore fix $\underline{b} = \underline{B}(S_n)$ and $p_1 = \ {}_np_1$ as exogenous parameters and require the distribution variables ${}_nX,\ {}_nA$ to satisfy the first-order KKT optimality conditions of the following problem:

$$\max_{{}_nX,\,{}_nA} \sum_{i=1}^{m(\underline{b})} {}_n\alpha_i(\underline{b} - \ {}_nx_i)\,{}_nx_i. \tag{28}$$

s.t.

$$\lambda_0:\quad 1 - \sum_{i=1}^{m(\underline{b})} {}_n\alpha_i \geq 0. \tag{28.1}$$

$$\lambda_1: \quad \left(\underline{b}-p_1\right)^{1-\rho} - \sum_{i=1}^{m(\underline{b})} {}_n\alpha_i(\underline{b} - {}_nx_i)^{1-\rho} = 0. \tag{28.2}$$

$$0 \leq {}_nx_1 < {}_nx_2 < \cdots < {}_nx_{m(\underline{b})} \leq \underline{b}. \tag{28.3}$$

$${}_n\alpha_1,\ {}_n\alpha_1, \ldots,\ {}_n\alpha_1 > 0. \tag{28.4}$$

With λ_0 and λ_1 as the Lagrange multipliers, we write

$$\begin{aligned} L\left({}_nX,\ {}_nA; \lambda_0, \lambda_1\right) \\ &= \sum_{i=1}^{m(\underline{b})} {}_n\alpha_i\left(\underline{b} - {}_nx_i\right) {}_nx_i + \lambda_0\left(1 - \sum_{i=1}^{m(\underline{b})} {}_n\alpha_i\right) \\ &+ \lambda_1\left(\left(\underline{b}-p_1\right)^{1-\rho} - \sum_{i=1}^{m(\underline{b})} {}_n\alpha_i(\underline{b} - {}_nx_i)^{1-\rho}\right). \end{aligned} \tag{29}$$

We are interested in the following two necessary conditions:

$$\frac{\partial L(\cdot)}{\partial\, {}_nx_i} = {}_n\alpha_i(\underline{b} - 2\,{}_nx_i) + \lambda_1\, {}_n\alpha_i(1-\rho)\left(\underline{b} - {}_nx_i\right)^{-\rho} = 0 \quad i = 1,2, \ldots, m\left(\underline{b}\right). \tag{30}$$

$$\frac{\partial L(\cdot)}{\partial\, {}_n\alpha_i} = (\underline{b} - {}_nx_i)\, {}_nx_i - \lambda_0 - \lambda_1\left(\underline{b} - {}_nx_i\right)^{1-\rho} = 0 \quad i = 1,2, \ldots, m\left(\underline{b}\right). \tag{31}$$

By the strict concavity of the utility function we have

$$\begin{aligned} \Delta\left(\underline{b}\right) &= \left(\underline{b}-p_1\right)^{1-\rho} - \sum_{i=1}^{m(\underline{b})} {}_n\alpha_i(\underline{b} - {}_nx_i)^{1-\rho} \\ &> \left(\underline{b}-p_1\right)^{1-\rho} - \left(\underline{b}\sum_{i=1}^{m(\underline{b})} {}_n\alpha_i - \sum_{i=1}^{m(\underline{b})} {}_n\alpha_i \cdot {}_nx_i\right)^{1-\rho}. \end{aligned} \tag{32}$$

By Lemma 1 we have $p_1 > \sum_{i=1}^{m(\underline{b})} {}_n\alpha_i \cdot {}_nx_i$. Since $\Delta\left(\underline{b}\right) = 0$ we get by equation (32) that $\sum_{i=1}^{m(\underline{b})} {}_n\alpha_i < 1$. Thus, the constraint (28.1) can never be binding at any optimum and $\lambda_0 = 0$.

Next, we assume without any loss of generality, that all ${}_n\alpha_i \in {}_nA$ are non-zero. (If any of the probabilities were zero, we could just drop the corresponding price variable and deal with the formS_{n-1}; we would then appropriately require $n \geq 4$).

From equations (30) and (31) we get:

$$\frac{2\,{}_nx_i - \underline{b}}{(1-\rho)\left(\underline{b} - {}_nx_i\right)^{-\rho}} = \frac{{}_nx_j}{\left(\underline{b} - {}_nx_j\right)^{-\rho}} \qquad \forall i = 1,2,\dots,m(\underline{b});\ \forall j = 1,2,\dots,m(\underline{b}). \tag{33}$$

However, this condition may possibly be satisfied by *exactly* two prices: $\frac{\underline{b}}{1-\rho}$ and $\underline{b}$. In order to see that, consider the following two functions

$$f_1(x) = \frac{2x - \underline{b}}{(1-\rho)\left(\underline{b} - x\right)^{-\rho}}. \tag{34}$$

$$f_2(x) = \frac{x}{\left(\underline{b} - x\right)^{-\rho}}. \tag{35}$$

Let now $y = \frac{\underline{b}}{1+\rho} + d$ where d is a constant. We evaluate the difference at this point:

$$f_1(y) - f_2(y) = d\left\{\underline{b} - \frac{\underline{b}}{1+\rho} - d\right\}\left\{\underline{b}\rho - d(1+\rho)\right\}. \tag{36}$$

When $y < \underline{b}$, both terms in curled brackets are strictly positive, and the two functions intersect if and only if $d = 0$. When$y \to \underline{b}$, both functions $f_1(y)$ and $f_2(y)$ approach zero, and the difference between them becomes arbitrarily small. This constitutes a contradiction of the optimality of policy S_n with any $n \geq 3$, so we conclude that the optimal policy with risk aversion entails charging *exactly* two prices, which completes our proof. □

Theorem 3: We begin with the formulation of the seller's (constrained) profit-maximization problem:

$$\max_{p_1,p_2,\alpha} p_1\left(1 - \frac{p_1 - \alpha^{\frac{1}{1-\rho}}p_2}{1 - \alpha^{\frac{1}{1-\rho}}}\right) + \alpha p_2\left(\frac{p_1 - \alpha^{\frac{1}{1-\rho}}p_2}{1 - \alpha^{\frac{1}{1-\rho}}} - p_2\right). \tag{37}$$

s.t.:

$$\lambda_1: \quad p_2 + k - 1 \geq 0. \tag{37.1}$$

$$\lambda_2: \quad p_1 - p_2 \geq 0. \tag{37.2}$$

We write

$$L(p_1, X, A; \lambda_1, \lambda_2)$$

$$= p_1\left(1 - \frac{p_1 - \alpha^{\frac{1}{1-\rho}} p_2}{1 - \alpha^{\frac{1}{1-\rho}}}\right) + \alpha p_2\left(\frac{p_1 - \alpha^{\frac{1}{1-\rho}} p_2}{1 - \alpha^{\frac{1}{1-\rho}}} - p_2\right) + \lambda_1\left(p_2 + k - 1\right)$$

$$+ \lambda_2\left(p_1 - p_2\right).$$

The set of first-order KKT necessary conditions comprises:

$$\frac{\partial L(\cdot)}{\partial p_1} = \frac{1}{1 - \alpha^{\frac{1}{1-\rho}}}\left(1 - 2p_1 + p_2\left(\alpha + \alpha^{\frac{1}{1-\rho}}\right) - \alpha^{\frac{1}{1-\rho}}\right) + \lambda_2 = 0. \tag{38}$$

$$\frac{\partial L(\cdot)}{\partial p_2} = \frac{1}{1 - \alpha^{\frac{1}{1-\rho}}}\left(p_1\left(\alpha + \alpha^{\frac{1}{1-\rho}}\right) - p_2 \alpha\right) + \lambda_1 + \lambda_2 = 0. \tag{39}$$

$$\frac{\partial L(\cdot)}{\partial \alpha} = \frac{p_1 - p_2}{\alpha\left(1 - \alpha^{\frac{1}{1-\rho}}\right)^2 (1-\rho)}\left(p_1 \alpha^{\frac{1}{1-\rho}} - p_2\left(\alpha(1-\rho) + \rho\alpha^{\frac{2-\rho}{1-\rho}}\right)\right) = 0. \tag{40}$$

$$\lambda_1(p_2 + k - 1) = 0. \tag{41}$$

$$\lambda_2(p_1 - p_2) = 0. \tag{42}$$

$$\lambda_1, \lambda_2 \leq 0. \tag{43}$$

Let us denote by $p_2^{unc}(\rho)$ the optimal solution for the second period price we obtained earlier for the uncapacitated case of Theorem 2. Note that the following holds for every ρ:

$$1 - p_2^{unc}(\rho) \geq \frac{1}{2} \geq \frac{1-\rho}{2-\rho} > 0 \quad , 0 \leq \rho < 1. \tag{44}$$

There exist three possible cases of capacity availability:

(i). $1 \geq k \geq 1 - p_2^{unc}(\rho) \geq \frac{1-\rho}{2-\rho}$.

(ii). $1 - p_2^{unc}(\rho) > k \geq \frac{1-\rho}{2-\rho}$.

(iii). $\frac{1-\rho}{2-\rho} > k \geq 0$.

We discuss each case separately.

Case (i): Obviously, the solution is the same as in the case with no production capacity constraint. The capacity in this case always exceeds the required threshold, and the monopoly profit is always higher than $\frac{1}{4}$, as required.

Case (ii) In this case we can *always* find a (two-price) policy that dominates any single price scheme. It suffices to show a solution that can be checked to verify this assertion for any parameter values:

$$S^*(\rho, k) = \{p_1^*, \{p_2^*, 1\}, \{\alpha^*, 1-\alpha^*\}\}.$$

$$p_1^* = \frac{1}{2} + \frac{\alpha^* + (\alpha^*)^{\frac{1}{1-\rho}}}{2}(1-k). \quad (45)$$

$$p_2^* = 1 - k. \quad (46)$$

where $\alpha^* \in (0,1)$ is given implicitly as the unique feasible solution to the equation

$$\alpha^{\frac{2}{1-\rho}}k + 2\alpha(1-k)(1-\rho) - \alpha^{\frac{1}{1-\rho}}\big(1 + \alpha(1-k)(1-2\rho)\big) = 0. \quad (47)$$

Case (iii) It remains to show that in this case there exists no policy that dominates the optimal one-price scheme that entails charging a price of $(1-k)$ in a single period. Let

$$S = S(\rho, k) = \{p_1, \{p_2, 1\}, \{\alpha, 1-\alpha\}\}$$

and assume that S is optimal and strictly dominates any one-price policy. In this case $p_1 > p_2$ and thus $\lambda_2 = 0$. Condition (38) yields

$$p_1 = \frac{1}{2}\left(1 - \alpha^{\frac{1}{1-\rho}} + \left(\alpha + \alpha^{\frac{1}{1-\rho}}\right)(1-k)\right). \tag{48}$$

By plugging the above expression into condition (39) we get,

$$\begin{aligned} &\frac{1}{2}\left(\alpha + \alpha^{\frac{1}{1-\rho}}\right)\left(1 - \alpha^{\frac{1}{1-\rho}} + \left(\alpha + \alpha^{\frac{1}{1-\rho}}\right)(1-k)\right) - 2\alpha(1-k) \\ &\qquad + \lambda_1\left(1 - \alpha^{\frac{1}{1-\rho}}\right) = 0 \end{aligned} \tag{49}$$

Let us rewrite this later condition as follows

$$\begin{aligned} &\frac{1}{2}\left(\alpha + \alpha^{\frac{1}{1-\rho}}\right)\left(1 - \alpha^{\frac{1}{1-\rho}} + \left(\alpha + \alpha^{\frac{1}{1-\rho}}\right)(1-k) - 2(1-k)\right) \\ &\qquad - 2\alpha(1-k) + \left(\alpha + \alpha^{\frac{1}{1-\rho}}\right)(1-k) + \lambda_1\left(1 - \alpha^{\frac{1}{1-\rho}}\right) = 0 \end{aligned} \tag{50}$$

By gathering terms we get:

$$\begin{aligned} &\frac{1}{2}\left(\alpha + \alpha^{\frac{1}{1-\rho}}\right)\left(-1 + \alpha + k\left(2 - \alpha - \alpha^{\frac{1}{1-\rho}}\right)\right) + \left(-\alpha + \alpha^{\frac{1}{1-\rho}}\right)(1-k) \\ &\qquad + \lambda_1\left(1 - \alpha^{\frac{1}{1-\rho}}\right) = 0 \end{aligned} \tag{51}$$

With the variables of the policy S, the second and the third terms on the left-hand side of condition (51) are negative. Therefore, for optimality to hold, the first term on the left-hand side of (51) must be positive:

$$k \geq \frac{1-\alpha}{2 - \alpha - \alpha^{\frac{1}{1-\rho}}}. \tag{52}$$

Whereas for any α and ρ ($0 \leq \alpha, \rho \leq 1$) it can be shown that,

$$\frac{1-\alpha}{2 - \alpha - \alpha^{\frac{1}{1-\rho}}} \geq \frac{1-\rho}{2-\rho}. \tag{53}$$

This is a contradiction to capacity falling under case (iii). Hence, there exists no policy that dominates the optimal one-price scheme when available capacity falls below the threshold defined in Theorem 3. □

<u>Theorem 4:</u> The result stems directly from the solutions for both uncapacitated and capacitated optimal policies as given by equations (23) to (25) and (45) to (47)}, respectively.

Figures:

Figure 1: Pricing Data For Flights Between London And New York

(Lowest available rountrip online fares* for departure on 2/18/2005 and return on 2/22/2005)

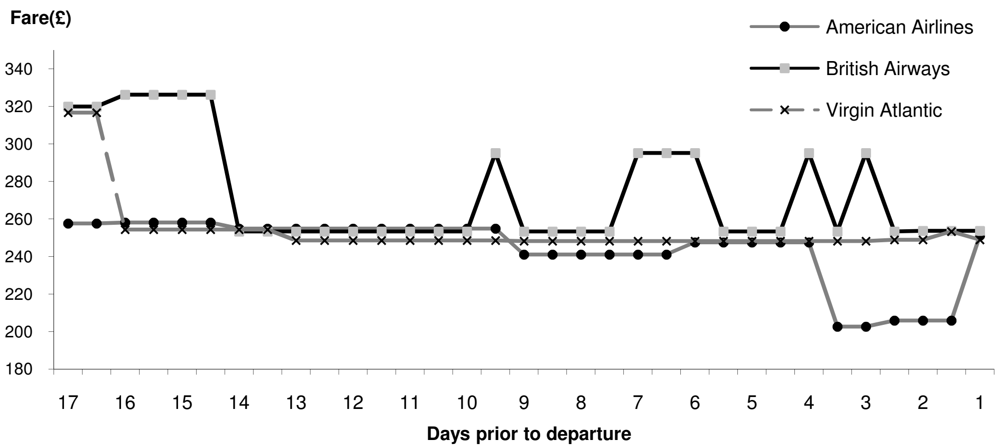

Figure 2: Market Segmentation Effects (Example)

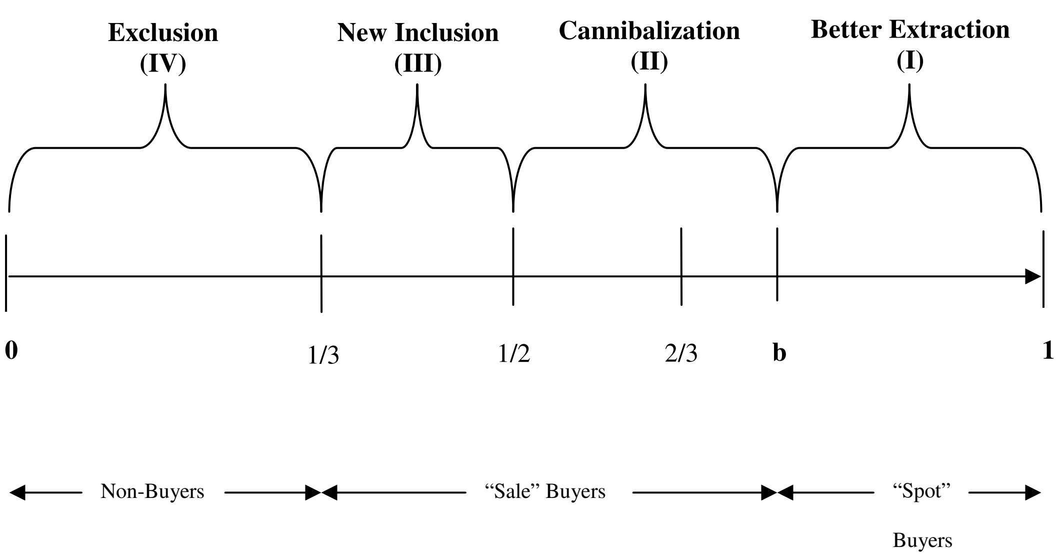

Figure 3: Unconstrained Model’s Behavior

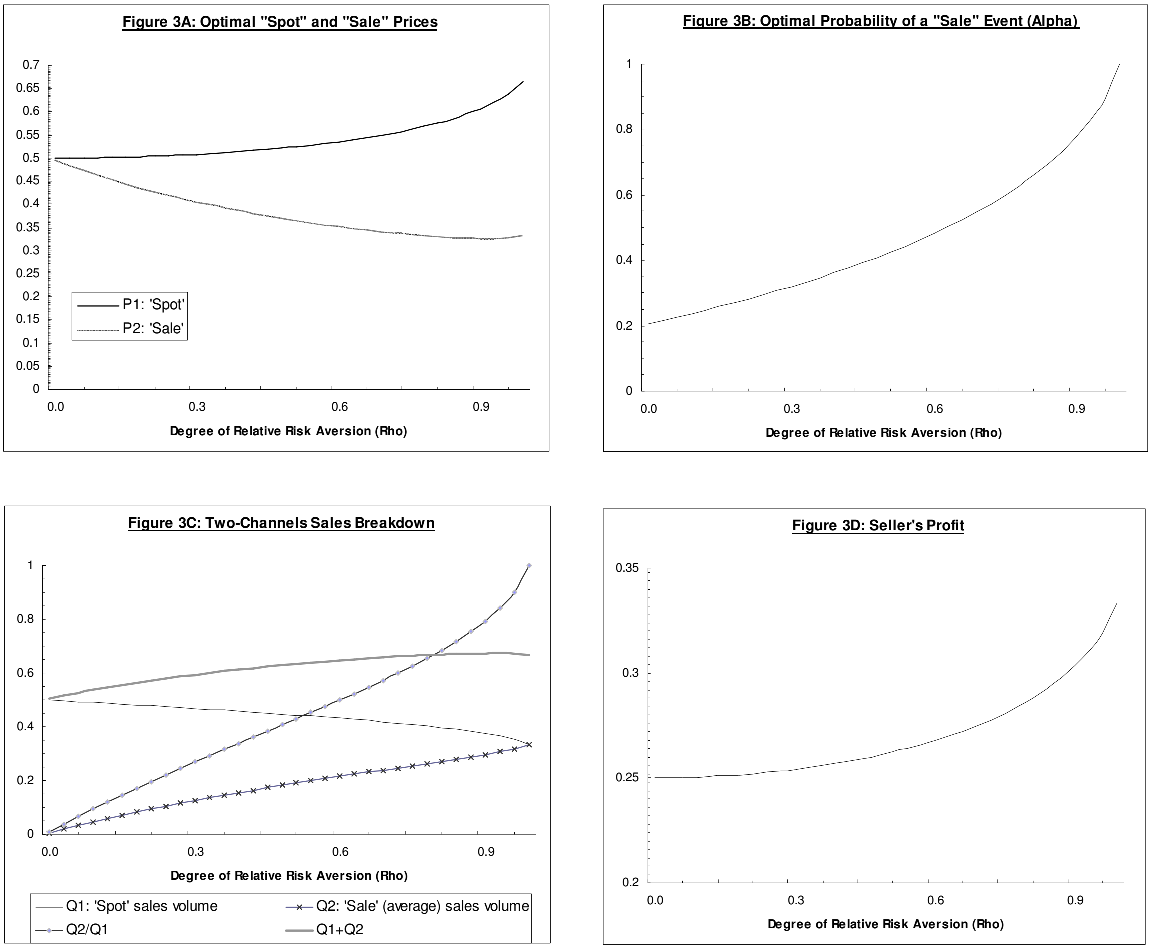

Figure 4: Optimal Pricing with Exogenous α

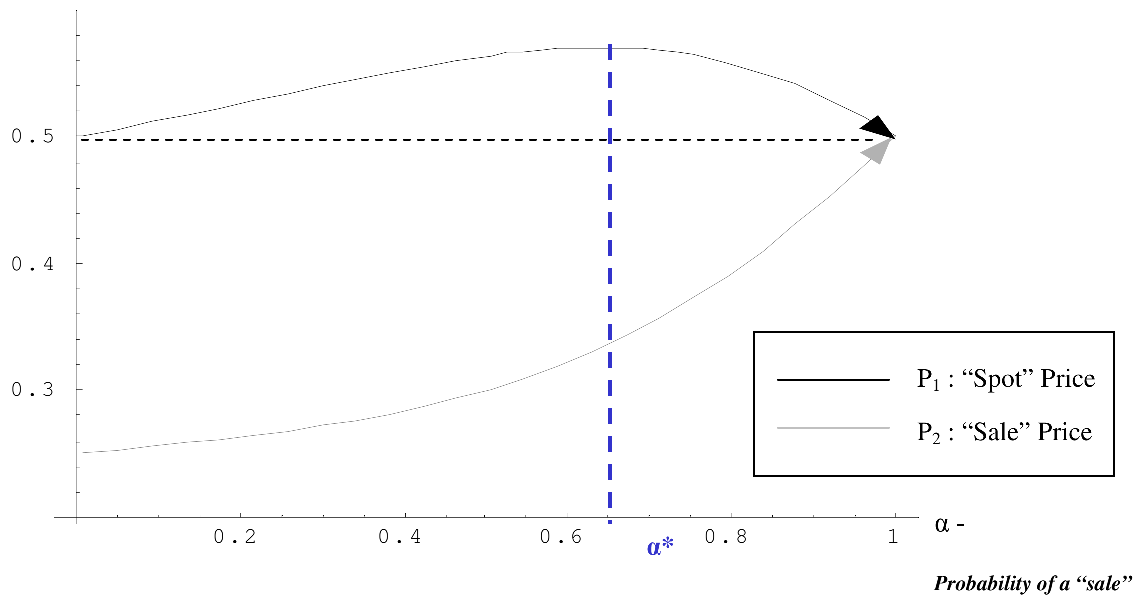

Figure 5: Two-Channel Sales Breakdown with Exogenous α

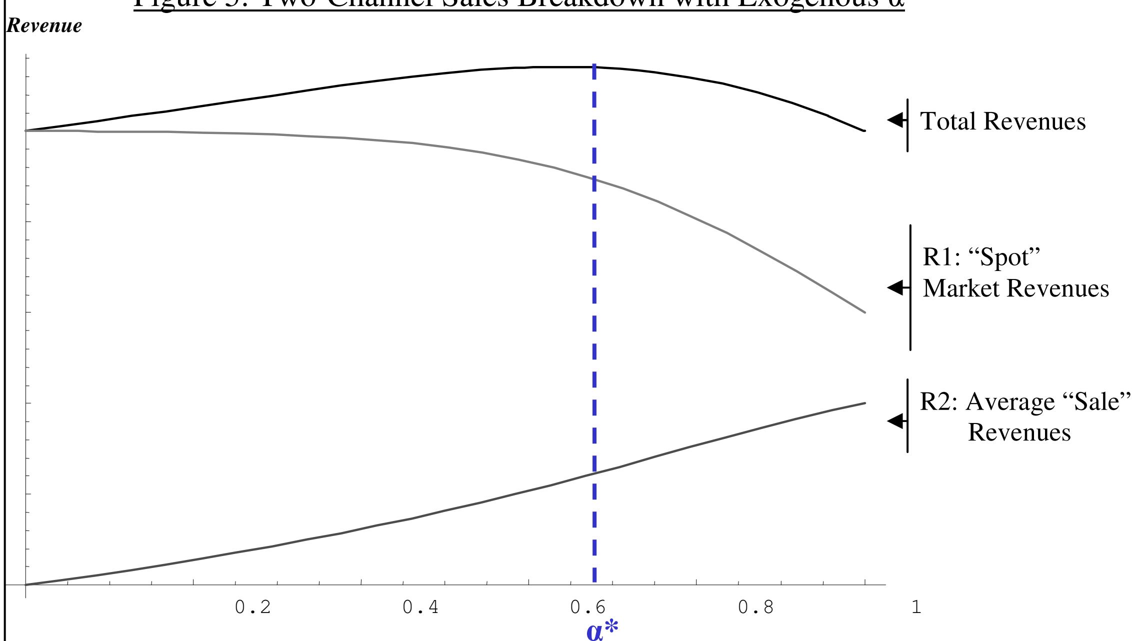

Figure 6: Feasibility of Segmentation under Capacity Constraint