March 24, 2011 Comments Welcome

Testing Factor-Model Explanations of Market Anomalies

Kent Daniel and Sheridan Titman*

- Abstract -

A set of recent papers attempts to explain the size and book-to-market anomalies with either: (1) conditional CAPM or conditional consumption-CAPM models with economically motivated conditioning variables, or (2) factor models based on economically motivated factors. The tests of these models use similar methodologies and similar test assets, and each test fails to reject the proposed model. This is surprising, as the correlation between the proposed factors is very small. We argue that many or all of these tests may fail to reject as a result of low statistical power. We propose an alternative test methodology which provides higher power against reasonable alternative hypotheses, and show that the new test methodology results in the rejection of several of the proposed factor models at high levels of statistical significance.

*Graduate School of Business, Columbia University and The College of Business Administration, University of Texas, Austin and NBER. emails: *kd2371@columbia.edu* and *titman@mail.utexas.edu*. We thank John Campbell, Kenneth French, Bruce Lehman, Martin Lettau, Lars Hansen, John Heaton, Robert Hodrick, Tuomo Vuolteenaho, James Weston and the participants of seminars at Columbia, CUNY-Baruch, Duke, Harvard Business School, the University of Maryland, Princeton, INSEAD, the Federal Reserve Bank of New York, Morgan Stanley and Goldman Sachs, and the participants of the NBER Asset Pricing Program Meetings, the IMA Financial Data Analysis and Applications Workshop, the Notre Dame Behavioral Finance Conference, the SQA Meetings, for helpful comments and suggestions. Finally, we thank Martin Lettau, Tuomo Vuolteenaho, and Ken French for the data used in empirical tests.

1 Introduction

The Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model suggests that the expected returns of risky assets should be determined by the covariance of their returns with the returns on the market portfolio. However, studies by Fama and French (1992) and others uncover almost no relation between market betas and expected returns, and instead find a strong cross-sectional relation between characteristics like size and book-to-market and returns. The nature of the underlying mechanism responsible for these empirical findings has now been a source of of debate for several decades, and there is still no consensus. One hypothesis is that a firm's size and book-to-market serve as proxies for the riskiness of the firm. Another possibility is that these characteristics proxy for mispricing, *i.e.*, high book to market stocks have higher expected returns because they are undervalued.

The results of Fama and French (1993) appear to provide support for the first hypothesis by showing that a set of factor-mimicking portfolios, MKT, SMB and HML, do a fairly good job of pricing the cross-section of stock returns. However, for several reasons this evidence is not particularly satisfying. First, the SMB and HML factors are not economically motivated; they are simply the returns from the financial assets that others conjecture are potentially mispriced. Indeed, Daniel and Titman (1997) argue that even if mispricing is responsible for the cross-sectional differences in returns, factor portfolios that are constructed in this way would still price size and book-to-market sorted portfolios.

Largely because of such concerns, a set of more recent papers have attempted to discover the underlying risks that might be responsible for the observed return patterns. If we abstract from liquidity and behavioral considerations, an asset's expected return is determined by the covariance between its realized returns and a representative agent's marginal utility, suggesting that the returns of the SMB and HML portfolios must in some ways capture the variation of more fundamental economic factors that are correlated with marginal utility. Researchers have investigated a number of such economically-motivated factor models.

We divide these models into two categories: *Conditional (C)CAPM Models* and *Alternative Factor Models*. The conditional versions of the CAPM and Consumption-CAPM (CCAPM) of Breeden (1979) retain the basic structure of the CAPM or CCAPM, but allow for time-variation in the covariation of asset returns with the market return (or consumption growth, in the case of the CCAPM), and time variation in the premium associated with this covariation. These models can be written as unconditional multifactor models where one factor is the market return, and the second factor is the market return interacted with a conditioning variable (see, *e.g.*, Harvey (1989), and later Cochrane (1996)). Similarly, a Conditional-CCAPM model can be expressed as a multi-factor model with factors equal to consumption growth and consumption growth interacted with a conditioning variable. Thus, the models effectively augment the market return (or consumption growth) with an additional factor equal to the scaled market or to scaled consumption growth.[1]

Alternative Factor Models propose a factor other than the standard market portfolio return or consumption growth as a pricing kernel. Some of the proposed models are unconditional (time-invariant), while others are conditional, meaning that the premium associated with the factor covariation, and the premium associated with this covariation are time varying. Based on the logic of Harvey (1989), such conditional factor models can be tested as scaled multifactor models with additional factors equal to the factors scaled by instruments which capture the time variation. Thus, both the Conditional (C)CAPM Models and Alternative Factor Models include additional factors beyond the standard market return (or consumption growth) that is missing from the standard (C)CAPM.

The models that have been proposed and tested are based on a number of plausible economic stories for why value stocks might be riskier than growth, and suggest economic factors that capture that risk. A subset of the papers that propose these tests is listed in Table 1. The tests in these papers generally fail to reject the proposed factor models,

[1]However, as with tests of any model, a rejection of a conditional factor model with a specification of this kind is a joint rejection of the factor model and the instruments used to capture the time-variation in the conditional covariance. If the instruments do not fully capture the time-variation the model may be incorrectly rejected.

suggesting that there are a number of plausible economic factors that can explain the value effect.

While at first glance these results appear promising, there are several reasons to question these findings. The first concern is that these results present a conundrum for anyone attempting to use the models. Which, if any, of these dozen or so models is the correct one to use in determining cost of capital? The results in these papers offer no answer to this question, as each of the proposed models appears to "work" reasonably well. If it were the case that each of the factor models would give about the same answer, this might not be a concern. However, the correlations between the proposed factors are actually very low, suggesting that the different models yield very different expected returns/costs of capital.

A second related concern is that asset pricing theory implies that there is a unique factor in the span of the asset return space that prices all assets.[3] This implies that the only way that two single factor models can each price the full cross-section of returns is if the projections of each of these factors onto the asset return space are equal. In other words, if the factors in two proposed single factor models both lie in the asset return space, then *at most one of the two models can be correct.*[4]

We argue here that the failure of these tests to reject so many different proposed factor models – with such low correlations between the proposed factors – is not because the data are supportive of the models. Instead, we argue, the culprit is the test methodology: all of these tests are done using portfolios – typically the 25 size and book-to-market portfolios first examined in Fama and French (1993) – and a Fama and MacBeth (1973)

[3]Following Hansen and Richard (1987), in incomplete markets multiple pricing kernels ($\tilde{m}$'s) may exist which price all assets, but there exists a unique projection of each of these pricing kernels onto the space of asset returns $\tilde{m}^*$. Following Hansen and Jagannathan (1991), this is equivalent to the statement that there is a unique mean-variance efficient portfolio that prices all excess returns.

[4]Yet another concern is raised by several studies that suggest problems with the proposed conditional (C)CAPM specifications. Lewellen and Nagel (2006) argue that the covariance of the conditional expected return on the market and of the conditional market betas of high and low book-to-market stocks is not high enough to explain the value effect. Also, Hodrick and Zhang (2001) find large specification errors for the Lettau and Ludvigson (2001) conditional CCAPM model. However, tests of these conditional CAPM models fail to reject the models, again suggesting the possibility that the failure to reject is a result of low test power.

like test methodology.[5]

There are two motivations for using test portfolios rather than individual stocks to test these models. The first is that it reduces the errors in variables problem that would otherwise plague the second stage Fama-MacBeth regression; since betas are estimated with error, the coefficients in the second stage regression are biased. A second problem has to do with the efficiency of the estimates. A more efficient GLS version of the Fama-MacBeth procedure requires an estimate of the covariance matrix, which creates challenges when the cross-section is larger than the time series. However, forming portfolios in the way that is done in these papers – on the basis of either *just* characteristics or *just* factor loadings – has the unintended consequence of inducing very high correlations between alternative factor loadings and the characteristics. As we show in Section 4, because of this, the tests have low power, that is the tests are unlikely to reject the proposed models even when the models are false.

The motivation for the use of book-to-market (BM) sorted portfolios as the test portfolios is also intuitively appealing: we know that there is a strong empirical relation between book-to-market and average returns. Intuitively, in constructing portfolios with a large dispersion in average returns, such sorts should, *a priori*, produce higher test power. However, BM is a "catch-all" variable, one that will proxy for sensitivity to a variety of macroeconomic innovations. For example, because growth firm values derive more from growth options, BM sorts will capture differential sensitivity to business cycle innovations.

Thus, book-to-market sorted portfolios are likely to produce both variation in expected returns and (correlated) variation in the loadings on any number of macroeconomic factors. Moreover, in grouping all of the assets with similar BM together, any variation in factor loading that is independent of BM is largely eliminated. The end result is that, even if the loadings on a proposed factor are only loosely correlated with the expected returns of the individual assets in the economy, the sorting procedure will result in a set of test portfolios which exhibit a strong relation between loadings on the proposed factor

[5] Note that generalized method of moments Hansen (1982, GMM) tests of moment restrictions across the 25 portfolios (for example) are still subject to the critique discussed in this paper.

and expected returns. The problem is that, in grouping all of the assets with similar BM together, any variation in factor loading that is independent of BM is washed out.

A slightly more technical way of seeing this is as follows: an asset pricing model will explain the average returns of a set of portfolios if and only if the pricing kernel implied by that model prices the test assets. Since the payoffs of any set of test assets will not span all sources of risk, there will not be a unique pricing kernel (a unique $\tilde{m}$). However, there is a unique pricing kernel that lies in the span of the payoffs/returns of the test assets (a unique $\tilde{m}^*$). Moreover, *any* model with an implied pricing kernel $\tilde{m}_i = \tilde{m}^* + \tilde{e}_i$, where $\tilde{e}_i$ is outside of the space spanned by the payoffs of the test assets, will properly price the set of test assets.

The problem with the use of the 25 size-BM sorted portfolios as test assets is that their payoffs lie in a low-dimensional subspace of the full payoff space. Specifically, Fama and French (1993) show that the average R^2 in time-series regressions of the returns of their 25 portfolios on their three stock market factors is 93%, and there is little variation in the loading on the market factor.[6] This means that, to a close approximation, the returns of the 25 FF portfolios lie in a two-dimensional subspace spanned by HML and SMB.

Many sources of economic risk can be expected to lie outside the span of the returns of these test assets, even if these sources of risk could be hedged using other portfolios of stocks. Moreover, if the risk-premium associated with each factor is left as a free parameter, as is generally done in the Fama and MacBeth (1973) procedure, any factor which is loosely correlated with the m^* implied by HML and SMB will appear to properly price these test assets, even if this model would not properly price a fuller set of assets.

This means that a powerful test requires that the test assets span a higher dimensional space. Specifically, the test assets should be augmented by portfolios which expand the space in the direction of the proposed factor, but which have no exposure to the underlying characteristics that – according to the alternative hypothesis – actually explain returns.

[6] See Table 6 of Fama and French (1993). Reported regression R^2s range from 83% to 97%. Reported loadings on $[RM(t) - RF(t)]$ range from 0.91 to 1.18.

To construct such portfolios requires the use of an instrument which is correlated with variation in loading on the proposed factor, and which is imperfectly correlated with book-to-market. In our empirical tests below, we use two sets of instruments: first, we use estimates of lagged betas on the proposed factors; second, we use industry membership. Industry portfolios exhibit variation in factor loadings relative to a number of macroeconomic factors but this variation is, to a large extent, unrelated to book-to-market ratios.

Using these instruments to form portfolios, we reexamine several of the models proposed in the literature. Based on this preliminary analysis, we argue that before any of these models can be accepted as an full explanation of the book-to-market effect more powerful tests, based on the framework we lay out here, need to be carried out.

The outline of the remainder of the paper is as follows. In Section 2 we discuss the conditional and unconditional tests and motivate the empirical design of the tests we discuss. In Section 4, we evaluate the power of the proposed tests both via some basic analytical results and a set of simulations. More formally, we do this by proposing an alternative hypothesis, and argue that this test methodology yields low power against this alternative. Second we propose a methodology that has higher statistical power. In Section 5, we apply this new methodology to test several recent alternative factor models, and find that these model are rejected at high levels of significance with the new methodology. Section 6 concludes.

2 The Equivalence of Conditional (C)CAPM Tests and Multi-Factor Models

In this section, we show that conditional CAPM models can be written as unconditional multifactor models where one factor is the market return, and the second factor is the market return interacted with a conditioning variable (Cochrane (2000)). Similarly, a conditional-CCAPM model can be expressed as a multi-factor model with factors equal to consumption growth and consumption growth interacted with a conditioning variable.

The intuition behind this argument is best seen via an example: suppose that, the market is conditionally mean-variance efficient, which means that for all assets i:

$$(R_{i,t+1} - R_{f,t+1}) = \beta_{i,t}(R_{m,t+1} - R_{f,t+1}) + \epsilon_{i,t+1}$$

where $\epsilon_{i,t} \perp 1, (R_{m,t+1} - R_{f,t+1})$. Taking (conditional) expectations of each side gives:

$$E_t[R_{i,t+1} - R_{f,t+1}] = \beta_{i,t} E_t[R_{m,t+1} - R_{f,t+1}].$$

This is the usual statement of the conditional CAPM. However, suppose also that the expected return on the market varies over time. There is now substantial evidence consistent with this: market returns are high in business cycle troughs and low at business cycle peaks.

Notice that, if value stocks have a high beta (i.e., if $\beta >> 1$) when the market's expected return is high, and a low beta ($\beta << 1$) when the market return is low, then the average or unconditional beta of the value stock could be close to 1, but the unconditional expected return of the value stocks would be much higher than the market's. Intuitively, the value stocks are effectively "market timing," taking on more risk when the reward to risk ratio (*i.e.*, the expected return on the market) is higher. Thus, if the CAPM were tested unconditionally, it would be rejected.

The same argument suggests that, if the consumption-CAPM holds, but the premium to consumption beta varies over time and the consumption beta of value stocks positively covaries with this premium, a test of the unconditional CCAPM will be rejected.

A remedy to this problem is to test a conditional version of the CAPM or CCAPM. This is typically done by assuming the asset's market beta is a linear function of a n-dimensional vector of instruments $\mathbf{Z}_t$ in the investors information set at t.[7] The restriction that is

[7] Again, an important caveat here is that, if the underlying assumption that the $\beta_{i,t} = \boldsymbol{\beta}_i' \mathbf{Z}_t$ is incorrect, the model may be incorrectly rejected

then tested is that the conditional loadings on the factors explains the average returns of the test assets, *i.e.*, that:

$$(R_{i,t+1} - R_{f,t+1}) = (\boldsymbol{\beta}_i' \mathbf{Z}_t)(R_{m,t+1} - R_{f,t+1}) + \epsilon_{i,t+1}$$

where the restriction that is now tested is that $E[\epsilon_{i,t+1} \cdot (R_{m,t+1} - R_{f,t+1}) \mathbf{Z}_t] = \mathbf{0}$.[8]

As has been pointed out in Cochrane (2000), this test is equivalent to a test of a multi-factor unconditional model in which the factors are the market, and a set of "scaled" market returns, that is:[9]

$$r_{i,t+1} = \beta_{1,i} \; r_{m,t+1} + \beta_{2,i}(r_{m,t+1} Z_{2,t}) + \cdots + \beta_{n,i} \; (r_{m,t+1} Z_{n,t}) + \epsilon_{i,t+1}$$

The important thing for us is that these tests are therefore equivalent to tests of the CAPM (or the CCAPM) plus one or more additional factors added on.

Thus, if the uncondtional CAPM fails and the conditional CAPM properly prices value and growth portfolios, it must be the case that it is covariation with the scaled market return that is responsible for the differences in expected return on the value and growth portfolios.

2.1 Conditional CAPM Model Tests

A number of (C)CAPM tests have now been proposed in the literature. One that has recently received a good deal of attention is that of Lettau and Ludvigson (2001, henceforth LL). LL argue that, based on the intuition behind the Campbell and Cochrane (1999) model, their *cay* variable should be a good proxy for the market risk premium.

In this paper, LL show that the betas of value/growth stocks are higher/lower when the

[8] Note that this is not the same as a *conditional* test of the conditional model. This would be a test of the restriction that $E[\epsilon_{i,t+1} \mathbf{Z}_t] = \mathbf{0}$ or that $E[\epsilon_{i,t+1}(R_{m,t+1} - R_{f,t+1}) \mathbf{Z}_t \otimes \mathbf{Z}_t] = \mathbf{0}$. See also Appendix A.

[9] Here, we are implicitly assuming that the first element of $\mathbf{Z}_t$ is one.

expected return on the market is high (*i.e.*, when cay is low). Even if value stocks have a lower unconditional beta than do growth stocks, their betas are much higher when the expected return on the market is higher. Thus, were one to test the unconditional CAPM on value/growth stocks, one could reject it. However, LL argue that once the conditional variation in beta and the expected return on the market is taken into account, the *conditional* CAPM and CCAPM do a good job of explaining these returns.

The methodology used by LL to test the conditional CAPM is similar to that used in many recent papers: they test whether the returns of the Fama and French (1993) 25 size/BM sorted portfolios can be explained by their conditional CAPM using Fama and MacBeth (1973) tests. They find that it does a very good job.

However, an alternative test of the LL conditional CAPM model is based on a Gibbons, Ross, and Shanken (1989, GRS) like time-series regression (as employed by Fama and French (1993)). For example, one can examine test whether the return of the Fama and French (1993) HML portfolio can be explained using the single regression:

$$\mathrm{HML}_t = \alpha + \beta_{vw} R^e_{vw,t} + \beta_{vwz}\, \widehat{cay}_{t-1} R^e_{vw,t} + e_t \tag{1}$$

For a conditional CAPM model such as this one, the interaction term $\widehat{cay}_{t-1} R^e_{vw,t}$ captures the extra return arising from the covariation of the HML beta with the expected return on the market, as discussed in the preceding subsection.

Using quarterly data over the period 1953:01-1998:04, the same period examined by LL, the estimated intercept ($\hat{\alpha}$) for this regression is 1.26%/quarter, ($t = 3.47$), which is both economically and statistically significant. For comparison, without including the $\widehat{cay}_{t-1} R^e_{vw,t}$ interaction term, the α is 1.50% (t=4.16). The difference between the intercept terms in the two regressions is 0.24%/quarter, which is not statistically different from zero. This simple regression dramatically illustrates the point argued by Lewellen and Nagel (2006): while the betas of value stocks do increase in economic downturns, they don't increase anywhere near enough to explain the high returns of the HML portfolio.[10]

This straightforward rejection of the LL model raises the question of why the tests reported in Lettau and Ludvigson (2001) appear to offer such strong support for their model? The answer, we argue in the next section, is that the ir tests fail to reveal evidence inconsistent with the model because their test methodology, which is used throughout this literature, has extremely low power.

3 Correlations of Candidate Factors

Table 2 shows the correlations of nine alternative factors utilized in models that have been proposed as potential explanations for the value premium, specifically those listed in Table 1. Among the factors, HML is the Fama and French (1993) value factor, r_m is the excess return on the CRSP value weighted index; Δc is the quarterly change in log nondurable and services consumption; Δy is the quarterly change in log income; Δ(prop) is the log-change in proprietary income; $\Delta \log(\alpha)$ is equal to consumption growth plus the change in the non-housing expenditure ratio (as in Piazzesi, Schneider, and Tuzel (2007)); and finally N_{CF} is the cash-flow news variable (Campbell and Vuolteenaho (2004)). Among the instruments, DP is the dividend-price ratio of the CRSP value-weighted index at the beginning of each quarter; $\widehat{cay}$ is the cay variable from Lettau and Ludvigson (2001); s is the labor income to consumption ratio of Santos and Veronesi (2005);

The correlations are all calculated on a quarterly basis. Each conditioning variable (DP, $\widehat{cay}$ and s) is demeaned. The sample correlations are each estimated using quarterly data over the period 1963Q4:1998Q3.

Interestingly, the correlation matrix shows that the correlations of each of the factors with HML is low – the highest is a 27% correlation with $-N_{CF}$, the principal factor of

[10] A simple test like this cannot be used to test the CCAPM, because the conditional MVE portfolio (which here will be the portfolio maximally correlated with consumption (Breeden (1979))) is not observable.

Campbell and Vuolteenaho (2004).[11] In addition, the correlations between the proposed factors are also for the most part quite small. Here, other than the correlations between DP, $\widehat{cay}$, and s interacted with the market, the maximum correlations between two factors is 28% (between the change in proprietary income and the log growth in the non-housing expenditure ratio.) Other than these the correlations are generally less than 20%.

4 Test Power for Characteristic Sorted Portfolios

There are now more than a dozen factor models that explain the returns of portfolios sorted on size and book-to-market. It can't really be the case that all of these factor models are "correct," in the sense that they all explain the cross-section of stock returns. Thus, our first task is to explain how so many different factors, with such a low average correlation, seem to explain the cross-section of returns.

As we discussed in the introduction, our explanation for this is that these tests are designed in such a way that the test lack statistical power. Specifically we argue that, under very weak conditions, *any* factor will appear to explain the average returns of size and book-to-market sorted portfolios. That is, these tests will make it appear that returns are consistent with the factor model even when they are not.

To illustrate this point, this section presents a simulation that demonstrates how the FF-FM methodology can provide spurious support for a factor model. This simulation is then used to motivate our approach for testing factor models against the characteristic alternative, an approach that we implement in our empirical tests in Section 5.

4.1 Simulation Results

The simulations presented here consider a factor that has been proposed to explain the observed cross-sectional relation between returns and characteristics. For example, in-

[11] See Section 5.3 for a discussion of the performance of this factor model, and in particular of this high correlation.

novations in housing price changes have been proposed as a factor that explains the book-to-market effect, which is known to be related to expected returns. To abstract from estimation problems we assume that we accurately measure both a firm's factor-beta and its expected return. In addition, we assume that factor betas are correlated with the characteristic.

In our simulations we randomly draw 2500 log book-to-market ratios and the single factor beta from a correlated normal distribution. Specifically, we draw from a multivariate normal distribution such that:

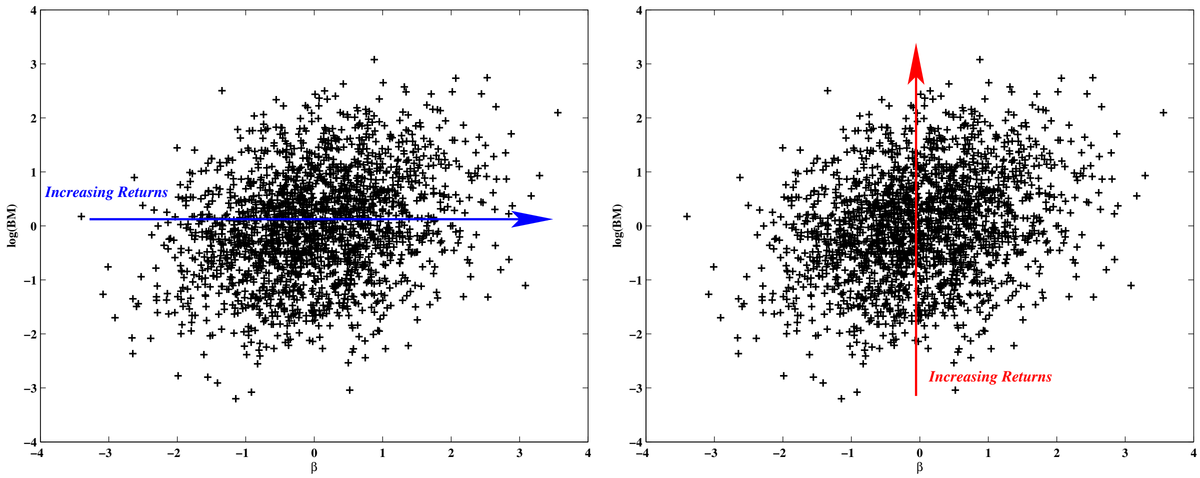
$$\begin{aligned} bm_i = \log(BM_i) &\sim \mathcal{N}(0,1) \\ \beta_i &\sim \mathcal{N}(0,1) \\ \rho(bm_i, \beta_i) &= \rho_{bm,\beta}. \end{aligned}$$

We assume a relatively weak correlation between the characteristic and the factor loading of $\rho = 0.3$, which is low enough to allow us to distinguish between the two hypotheses in an appropriately designed test. As we mentioned earlier, there are a number of good reasons why factors are likely to be correlated with book-to-market ratios. Theoretically, we know that a firm's book-to-market ratio is a good proxy for a firm's future growth (see, *e.g.*, Fama and French (1995), Cohen, Polk, and Vuolteenaho (2003)), and high- and low-growth firms are likely to have different sensitivities to a number of economic factors. Empirically, Table 1 shows that this is indeed the case.

Figure 1 illustrates the distribution of characteristics and factor loadings that are generated from the simulation. This figure has two plots. Consider the left side plot first. The vertical (y-axis) is the firm's log book-to-market ratio, and the horizontal (x-axis) is the factor beta. Each of the 2500 crosses in this figure represents a single firm or stock. The weak correlation can be seen in the distribution of the crosses: high β firms generally have high BM ratios, but there is considerable variation in βs that is unrelated to BM.

Figure 1 illustrates the null and alternative hypotheses we'll consider. The null hypothesis, that the factor model fully explains the cross-section of returns, is represented by the left

Figure 1: Null and Alternative Hypotheses



plot. Under the null, a stock's expected return increases with β (as you move to the right in the plot), but is unrelated to book-to-market *after controlling for beta.* Of course, there is still an unconditional relation between book-to-market and expected returns.

This null hypothesis is a risk-based story for the book-to-market effect: a firm's book-to-market ratio forecasts its future return because it serves as is a proxy for systematic risk. Under this hypothesis, if the factor loading can be observed, it will better explain average returns than book-to-market ratios.

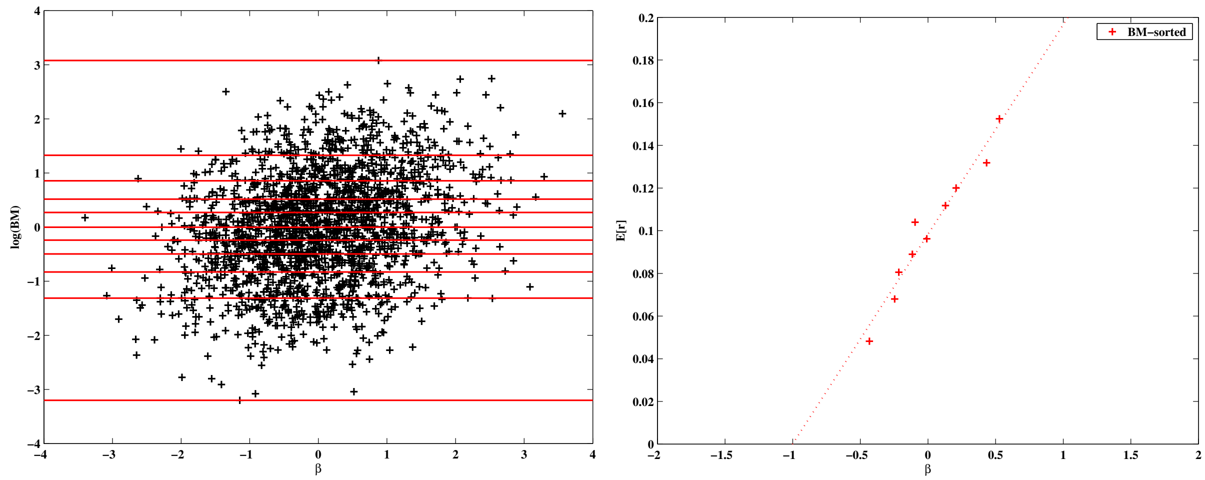
A test's power is defined as the probably of the test's rejecting the null hypothesis given that the alternative is true. Thus, test power can only be evaluated relative to an alternative hypothesis. The alternative hypothesis we propose is illustrated in the right hand side plot in Figure 1. Under the alternative, the expected return is linearly related to the log book-to-market ratio, but is not directly related to the factor beta. That is, the beta is related to returns only through its correlation with the characteristic.

To evaluate power of the tests, we calculate expected returns under the alternative hypothesis; that is:

$$E[r] = \lambda_0 + \lambda_1 \log(B/M).$$

Following, the test procedures used in the literature, we then sort the 2500 (simulated) firms into 10 portfolios depending on their book-to-market ratios. This sort is illustrated

Figure 2: BM-Sorted Portfolio Formation



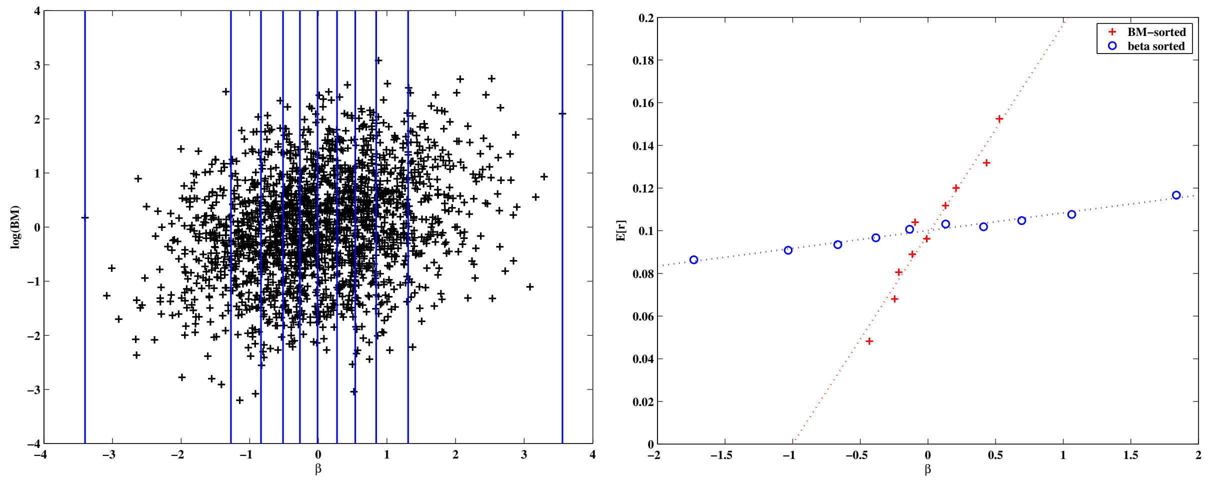
in the left panel of Figure 2. Each horizontal line in the figure represents the cutoff between the BM deciles: the number of firms between any two lines is 250 (one-tenth of the sample).

This figure illustrates the problem that arises when characteristic sorted portfolios are used to test factor models. Notice that the top decile will have a high average BM ratio, and will therefore have a high expected return. In addition, it will have a high factor beta as a result of the correlation between beta and BM. As we move from the top to the bottom BM decile, the average return and the average (portfolio) beta declines.

Note also, that there is very little variation in betas in the different deciles, since differences in the betas that are not correlated with the characteristic is "diversified away" by the portfolio formation procedure. As a result, as we show in the right panel of Figure 2, which plots the expected returns and betas for these 10 portfolios, these variables are very highly correlated. The regression R^2 here is 94.4%. The reason is that for this set of characteristic-sorted portfolios, there is almost no independent variation in beta.

One can alternatively sort stocks into portfolios based on risk rather than characteristics. Figure 3 illustrates this formation method. In the left-hand plot the vertical lines show the cutoffs between beta-sorted deciles. The right side figure plots the expected returns and betas of these portfolios, and the regression line relating these two. The corresponding

Figure 3: Beta-Sorted Portfolio and Cross-Sectional Test



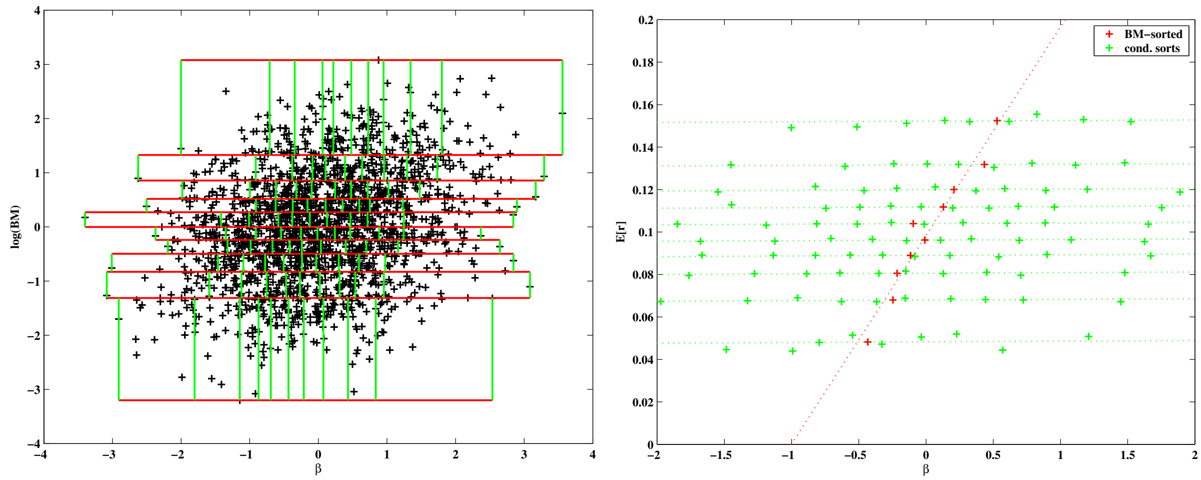
regression line for the BM sorted portfolios is also shown.

This plot shows that sorting portfolios in this way results in a lower estimated factor risk premium, but still yields a strong estimated relation between risk and return under the alternative, and again a good model fit, with a regression R^2 of 97.4%. Here, the problem is that high beta portfolios have, on average, high BM ratios and therefore high returns. Sorting in this way, there is almost no independent variation in BM across portfolios so that the betas and the characteristics are again almost perfectly correlated, making it impossible to discriminate between the two hypotheses.

In order to discriminate between to the two hypotheses one must construct test portfolios in a way that significant independent variation in betas and BM ratios. Figure 4 shows how this can be done with a multiple sort procedure.

The left panel of Figure 4 is similar to the corresponding panel in Figure 2: the horizontal lines again show the bounds of the 10 BM-sorted portfolios. Now, however we have superimposed on the plot a set of vertical lines which indicate the result of splitting each of the BM decile portfolios into 10 sub-portfolios based on beta; each of the 100 BM/beta sorted-portfolio contains 25 firms. With these portfolios there is substantial independent variation in beta, and therefore it is possible to discriminate between the null and alternative hypotheses.

Figure 4: Characteristic/Beta Sorted Portfolio Test



The results of the regression tests on the three sets of portfolios are summarized in Table 3. Notice that the R^2s are high in each of the three tests. The R^2 is clearly not a good indicator of model fit. Notice also that, for the test on either the BM-sorted or β-sorted portfolios, the estimated risk premia are large and highly significant. Only when the test is done with the multiple-sort portfolios, and when dummy variables are included for the BM ranking, does the premium get close to the true value of zero. Interestingly, even in the third test, the estimated premium is still significant in the simulation. The reason is that, within each of the 100 portfolios, there is still some correlated variation in expected return and book-to-market.

5 Empirical Tests

We now reexamine the results of three tests factor models. In each case we modify test by expanding the set of test assets as discussed in Section 4.

In Section 5.1, we revisit tests of the CAPM in the 1929-1963 period. Several recent tests have failed to reject the CAPM over this sample period – we argue as a result of low test power. We rexamine the performance of the CAPM with a test which remedies the low test power and find that we can comfortably reject the CAPM in this pre-1963 sample period.

Sections 5.2 and 5.3, examine tests of the Campbell and Vuolteenaho (2004) model in the post-1963 time period. Over this sample, they find that while a standard CAPM cannot explain the pattern of average returns, their model with cashflow and discount-rate betas can successfully explain the return patterns observed in this period.

In Section 5.4, we explore an alternative method of increasing test power – specifically by augmenting the asset return space utilizing industry portfolios. We apply this approach to test the model of Lettau and Ludvigson (2001).

5.1 The Performance of the CAPM over the Pre-1963 Period

Campbell and Vuolteenaho (2004) and Ang and Chen (2007) argue that the standard CAPM is not rejected over the 1929-1963 period. In contrast with the post-1963 period in which the market betas of value stocks are relatively low, in the 1929-1963 period the betas of value stocks are large and consistent with the higher returns earned by value stocks over this period. In formal tests both papers fail to reject the CAPM over the 1929-1963 time period.

We begin our empirical analysis by re-examining the data from this period using portfolios which employ our sort methodology. Table 4 shows the average returns and post-formation betas for our portfolios. Our portfolio formation procedure closely follows Daniel and Titman (1997). We form 45 portfolios using the following procedure: First, we sort all firms into three portfolios on the basis of market capitalization (or size) as of December of year t, on the basis of NYSE breakpoints. Additionally, we sort firms into three portfolios on the basis of the firm's book to market ratio. The book to market ratio is defined as the ratio of the firm's book value at the firm's fiscal year end in year t, divided by the firm's market capitalization as of December of year t.

Then, we sort each of the firms in these nine portfolios into five sub-portfolios based on the estimated pre-formation market betas (β_{mkt}) of these portfolios. We estimate the pre-formation betas by running regressions of individual firm excess monthly returns on the excess monthly returns of the CRSP value weighted index for 60 months leading up to December of year t. Sub-portfolio breakpoints are set so that, across each size-BM portfolio, there are an equal number of firms in each sub-portfolio.[12]

We then construct the realized returns for each of these 45 test portfolios. Even though the portfolios are formed using data up through the end of year t, we examine the returns from these portfolios starting in July of year $t+1$. The reason for this (following Fama and French (1993)) is that the book value data for the firm is unlikely to be publicly available as of January of year $t+1$, but it is almost certain to be available as of July. All of our portfolio returns are value-weighted. The portfolios are then rebalanced at the start of July of year $t+2$ using the new firm data from the end of year $t+1$.

The upper panel of Table 4 gives the average returns and t-statistics for each of our 45 portfolios. The lower panel gives the estimated post-formation betas for the realized returns, and the t-statistics associated with these betas. The final row of each of the two tables gives the average return/beta and the associated t-statistics for the "average portfolio," which is an equal weighted portfolio of each of the nine sub-portfolios in the same pre-formation beta group.

Finally, the last two columns of each table give the average return/beta and the associated t-statistics for the $5-1$ difference portfolio: that is a zero investment portfolio which buys one dollar of the high-estimated beta portfolio and shorts one dollar of the low-estimated beta portfolio.

A couple of important features of the data are evident in this Table. First, in the lower panel, in the two rightmost columns, note that our sort on pre-formation beta produces an economically large and highly significant spread in post-formation (realized) betas. This is important for the power of the test.

In contrast, the rightmost columns in the upper panel of Table 4 show that the sort on preformation beta produces little spread in average returns. The average return and t-

[12]However, note that the number of firms in the size-BM portfolios will vary because of the use of NYSE breakpoints in the size portfolio sort.

statistic in the lower right corner of this panel shows that the mean return differential between the high beta and the low beta portfolios is only 0.02% per month. The nine entries directly above this show that the differences in beta don't produce a statistically significant difference in return for any of the nine size/BM portfolios.

It is important to note that the observation of Ang and Chen (2007) and Campbell and Vuolteenaho (2004) that higher book to market is associated with higher beta in his early period is confirmed in our test: the lower panel of our table shows that, on average, higher book to market firms and smaller firms do indeed have higher betas: consistent with the reasoning laid out here, this positive correlation implies that market beta "explains" returns fairly well for size/B-M sorted portfolios. However, this positive correlation does not imply that the CAPM explains the returns of the full cross-section of common stocks in this period. When we expand the set of portfolio so as to capture variation in beta unrelated to variation in size or book to market, we see that differences in beta that are independent of differences in book to market are not associated with average return differences, at least at any statistically significant level.

We formally test the hypothesis that the CAPM explains the returns of these 45 portfolios by running time series regressions of the realized excess returns of the portfolios on the realized excess returns of the CRSP value-weighted portfolio returns over the 1929:07-1963:06 period. That is, the regressions are of the form:

$$(\tilde{r}_{i,t} - r_{f,t}) = \alpha_i + \beta_i(\tilde{r}_{m,t} - r_{f,t}) + \tilde{\epsilon}_{i,t}$$

The left part of Table 5 reports the estimated regression intercepts, and the right part presents the t-statistics associated with these intercepts. The last row of the table gives the intercepts and t-statistics for the average portfolio, and the last two columns give the estimated intercepts and t-stats for the 5−1 difference portfolio. Consistent with the average returns and estimated betas reported in Table 4, the estimated alphas are negative, are economically large, and are highly statistically significant. Indeed, for the average 5–1 difference portfolio (the lower-right entry in the table), the t-statistic is -4.42, strongly rejecting the single-factor CAPM in this time period.

5.2 Description of the Campbell and Vuolteenaho model

Campbell and Vuolteenaho (2004) propose a version of the Merton (1973) Intertemporal CAPM as an alternative to the static CAPM, and present empirical evidence showing that this model can explain the size/book-to-market anomaly. Their model relies on the decomposition of the realized market return into three components: (1) the conditional expected return ($E_t[r_{t+1}]$); (2) the component of the return attributable to news about the level of future cash flows ($N_{CF,t+1}$); and (3) return component attributable to news about the discount rates applied to these cash flows by investors ($-N_{DR,t+1}$):

$$\tilde{r}_{t+1} = E_t[r_{t+1}] + \tilde{N}_{CF,t+1} - \tilde{N}_{DR,t+1}. \tag{2}$$

Based on this decomposition, Campbell and Vuolteenaho (2004) estimate βs with respect to the two return components. To understand the motivation of these empirical tests, it is helpful to start with the representative agent model of Campbell (1993), which expresses expected returns as

$$E_t[r_{i,t+1}] - r_{r,t+1} + \frac{\sigma_{i,t}^2}{2} = \gamma\sigma_{p,t}^2\beta_{i,CF_p,t} + \sigma_{p,t}^2\beta_{i,DR_p,t},$$

where p is the portfolio the representative investor chooses to hold (*i.e.*, the market), γ is the coefficient of relative risk aversion of the representative agent, and $\beta_{i,CF_p,t}$ and $\beta_{i,DR_p,t}$ are the components of portfolio p's return attributable to cash-flow and discount-rate news, respectively. It is clear from this equation that if $\gamma = 1$ (*i.e.*, with a log-utility representative agent) the standard CAPM will obtain, as the covariance with changes in the investment opportunity set don't affect expected returns, consistent with Merton (1973). However. for a non-myopic representative investor, the premium associated with cash flow and discount rate betas will differ. Since we generally believe that investor preferences are characterized by $\gamma > 1$, the premium associated with cash flow risk should be greater than that associated with discount-rate risk.

The insight underlying the Campbell and Vuolteenaho (2004) model is that since growth stocks have longer duration, their returns should be more sensitive to changes in discount rates. Moreover, following Merton (1973), for reasonable preferences, covariance with discount rate shocks is likely to command a far lower premium than a high covariance with cash flow shocks. Thus, even though growth stocks have a higher market beta, most of this is β_{DR}, which has a low premium, so growth stocks (rationally) require a lower risk premium. In contrast, while value stocks have lower market-beta, most of their beta comes from β_{CF}, which commands a high premium. Hence value stocks (rationally) require a high risk premium.

As noted above, Campbell and Vuolteenaho (2004) find that over the 1929-1963 period, the standard CAPM prices the 25 Fama-French portfolios. Their interpretation of this observation is that value stocks have both a higher β_{DR} and a higher β_{CF} over this sample period, and as a result, there is a high correlation between the CAPM single β and its two components. However, over the 1963-2001 sample, the value stocks have a lower β_{DR}, but a higher β_{CF}. The measured CAPM β, which is the sum β_{DR} and β_{CF}, is lower for value stocks, but the average returns are higher because of the much higher premium attached to cash-flow risk. This, they argue, leads to a rejection of the standard CAPM. However based on their empirical tests they conclude that a model which allows for distinct premia for β_{DR} and β_{CF} is not rejected.

To empirically decompose beta into its cash flow and discount rate components, Campbell and Vuolteenaho (2004) first estimate discount rate shocks. To do this they use a number of variables that forecast future market returns in a vector-autoregression (VAR). One of these variables is their *value spread* – the difference between the average book-to-market ratio of the Fama and French (1993) small-value and small-growth portfolios:

$$\mathrm{VS}_t = \log\left(\frac{B_t}{M_t}\right)_{SmVal} - \log\left(\frac{B_t}{M_t}\right)_{SmGro}. \tag{3}$$

The reason for the inclusion of this variable is that this definition of the value-spread does a good job of forecasting the market return. However, given the inclusion of this variable in the VAR process and the methodology employed, the estimated N_{CF} is strongly positively correlated with the returns to HML. Given Campbell and Vuolteenaho (2004)'s estimated parameters:

$$N_{CF,t} = 0.004 + 0.60\ R_{m,t} + 0.40\ R_{m,t-1} + 0.01\ \Delta PE_t - 0.88\ \Delta TY_t\ - 0.28\ \Delta VS_t \quad (4)$$

where Δ is the one period change in the variable. Given the definition of the value-spread in equation (4),

$$-0.28\ \Delta VS_t = +0.28 r_{SH} - r_{SL}.$$

That is, the measured innovations in cashflow are positively linked to the difference between the returns of small-value and small-growth firms, and consequently are also strongly correlated with HML. This is important in understanding why β_{CF} "picks up" the value premium.

5.3 Tests of the Campbell and Vuolteenaho (2004) Model

Campbell and Vuolteenaho (2004) show that both over their full 1929-2001 period, and over two sub-samples (1929-1963 and 1963-2001), their model can explain the returns of the 25 size and values sorted portfolios, and in particular their model is not rejected in the post-1963 period where the CAPM fails. Over this period, value stocks have a lower β_{DR}, but a higher β_{CF}. The measured CAPM β, which is the sum of β_{DR} and β_{CF}, is lower for value stocks, but the average returns are higher because of the much higher premium attached to cash-flow risk. This leads to a rejection of the standard CAPM. However, they argue, a model which separates out the betas with discount-rate and cash-flow shocks is not rejected for the 25 FF size and book-to-market sorted portfolios.

In Subsection 5.1, we showed that with our 45 portfolios sorted on size, book-to-market, and *ex-ante* market-beta, we were able to reject the CAPM over this sample period. We begin our empirical tests here by examining the ability of the Campbell and Vuolteenaho

(2004) model to explain the cross-section of returns in the pre-1963 period, using the same set of portfolios.

The results from these tests of the Campbell and Vuolteenaho (2004) model over this period are reported in Table 6.[13] It is the numbers in the upper panel that are most relevant here. First, notice that there is indeed a large correlation between book-to-market and the cash-flow beta over this period: high BM firms do, on average, have higher cash-flow betas. However, sorting on pre-formation market beta also produces a large spread in cash-flow betas (note the t-statistics in the last column of the table), and as we have already seen in Table 4, produces no statistically significant spread in returns. Thus, differences in β_{CF} that are unrelated to differences in BM are not priced, contradicting the premise of the model.

To examine the model in the post-1963 period, we perform sorts on the basis of *ex-ante* estimates of firm's loadings on the cash-flow betas. We perform the sorts in this way because, in the post-1963 period, sorting on pre-formation market beta produces a large spread in the discount rate beta in the later part of the sample, but no statistically significant differences in cash-flow betas.

To estimate cashflow-betas, we first form a time series of estimates of cashflow-innovations. We sort on a linear combination of the variables that Campbell and Vuolteenaho (2004) use in their vector autoregression to estimate cash-flow news. Specifically, as shown in equation (4), their VAR estimates of N_{CF} are equivalent to a linear combination of the RHS variables of their VAR. We therefore calculate our pre-formation estimates of β_{CF} by regressing individual firm excess returns on $N_{CF,t}$ in the 60 months leading up to December of year t (consistent with our method as described in Section 5.1). Other details of the portfolio formation procedure are as described in Section 5.1.

Table 7 presents the post-1963 post-formation CF and DR betas and the associated t-statistics. Consistent with the Campbell and Vuolteenaho (2004) results, we find that a

[13]The data on the innovations N_{CF} and N_{DR} used in the empirical tests in this section are courtesy of Tuomo Vuolteenaho.

higher BM ratio is generally associated with a higher $\hat{\beta}_{CF}$, but with a lower $\hat{b}eta_{DR}$. Also, there is variation in both $\hat{b}eta_{DR}$ and $\hat{\beta}_{DR}$ (in the same direction) associated with average firm size.

Table 7 presents the post-1963 post-formation CF and DR betas and the associated t-statistics. Here, the spread is smaller than what is achieved in the early period sort on CAPM beta, but it is still statistically significant. Moreover, the CF beta spread in the 5−1 portfolio is about as large as the spread in CF beta achieved via unconditional sorts on book-to-market: the lower right corner of the upper panel shows a spread of 0.068, with a t-statistic of 3.54.

However, while the sort produces a statistically significant difference in post-formation cash-flow betas, it produces no statistically significant difference in average returns. Table 8 reports the average returns of the 45 late-period portfolios. The mean return of the average difference portfolio is 0.09, with a t-statistic of 0.054.

5.4 Testing the Lettau and Ludvigson (2001) Model with Industry Portfolios

As we noted in the introduction, researchers test their models with portfolios rather than individual stocks to avoid an errors-in-variables problem and to lower the dimensionality of the covariance matrix of returns. The advantage of using characteristic-sorted portfolios is that their returns exhibit a large spread in both factor loadings and realized returns. Of course, if a proposed factor model is correct, portfolios that generate a large spread in factor loadings should generate a spread in realized returns.

In this section we re-examine the Conditional Consumption-CAPM (CCAPM) test of Lettau and Ludvigson (2001). First, we reproduce their Fama MacBeth tests (in their Table 3), but use industry-sorted portfolio returns as our test assets rather than the characteristic-sorted portfolio returns that they use in their paper.[14] The reason for

[14]The returns to the sets of industry portfolios are based on the portfolios documented in Fama and

augmenting the test assets with industry portfolios is that, as we demonstrate below, industry-sorted portfolios exhibit a relatively large spread in the loadings on the Lettau and Ludvigson CCAPM factor $\widehat{\text{cay}}_t \Delta c_{t+1}$. Recall that what we want, in terms of augmenting the factor space, is a set of assets with disperse loadings on the proposed new factors – here $\widehat{\text{cay}}_t \Delta c_{t+1}$ – which is unrelated to book-to-price. The industry portfolios appear to meet this criterion.

Table 9, which presents the results of these sets of Fama-MacBeth regressions, indicates that the premia on $\widehat{\text{cay}}_t \Delta c_{t+1}$ is not significantly different from zero for the industry portfolios. Moreover while the R^2 is very high for the size/BM sorted portfolios, it is low for the FM regressions using the 38 and 48 industry portfolios. For the regression with the 11 FF industries, the R^2 is 51%, but the sign of the premium on scaled consumption growth is reversed.

In addition, we reproduce Figure 1, Panel (d) of Lettau and Ludvigson (2001), here for the Fama and French 38 industry sorted portfolios. The left panel of Figure 5 is done per the their methodology, and using the 25 FF test portfolios. Consistent with Lettau and Ludvigson's results, we find that the conditional consumption CAPM does a good job pricing this set of test assets.

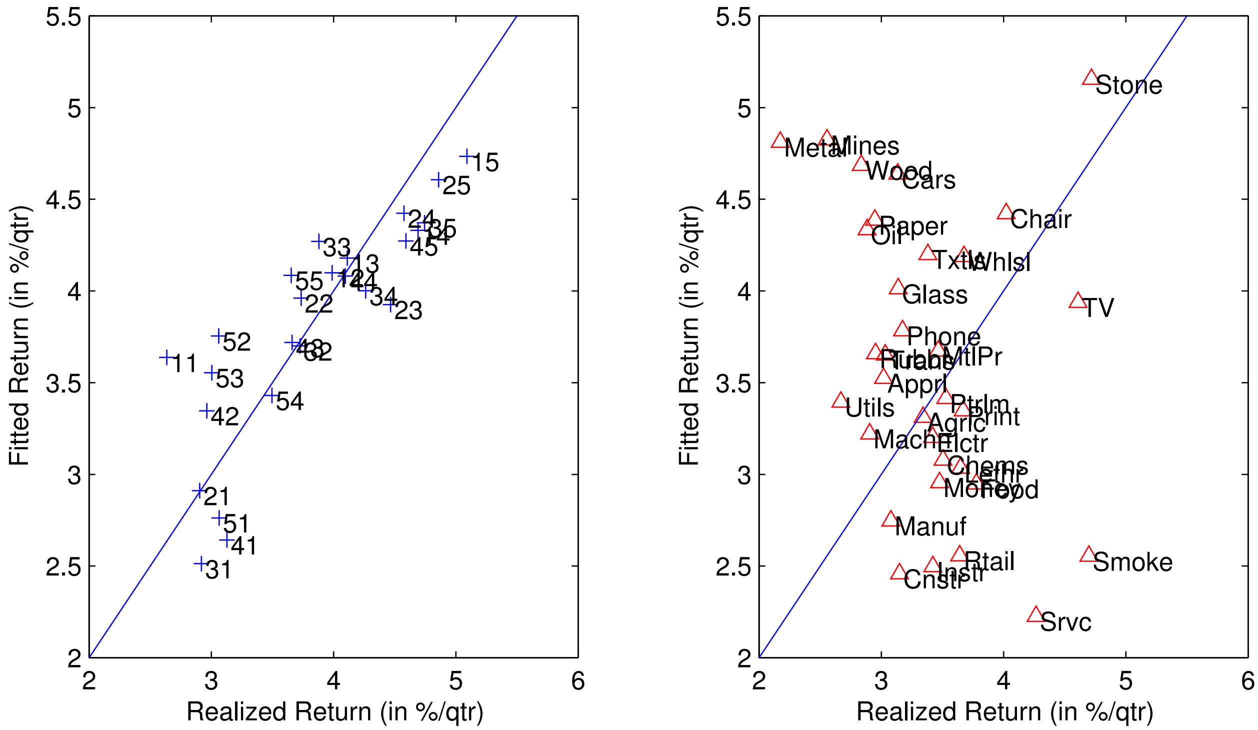
However, the right panel of the figure plots the fitted and realized returns for the 38 Fama and French industry portfolios. However, in constructing the fitted returns, we use the risk premia as estimated from the Fama-MacBeth regressions with the 25 sz/bm-sorted industry portfolios. Like the 25 Size/BM sorted portfolios, the industry sorted portfolios exhibit considerable variation in their loadings on the factors, and consequently very different fitted returns. However, the premia as estimated from the original test assets are not consistent with the pricing of the industry portfolios.

This empirical analysis suggests that the covariation with the proposed factors outside of the original Fama/French-25 portfolio return space is not priced is a manner consistent

French (1997), and are taken from Ken French's web page. Details on the SIC codes associated with each of the industry breakdowns is available there. The factor data – $\widehat{cay}_t$, Δc_{t+1}, and $\widehat{cay}_t \cdot \Delta c_{t+1}$ – are courtesy of Martin Lettau, and are the returns examined in Lettau and Ludvigson (2001).

Figure 5: **Lettau and Ludvigson (2001) Conditional CCAPM Model – Realized and Fitted Returns with Alternative Test Assets.**

This figure presents the fitted and realized returns for (1) the 25 size-book-to-market sorted portfolios of Fama and French (1993) in the left panel, and (2) the 38 industry portfolios from Fama and French (1997) in the right panel. As in Lettau and Ludvigson (2001), the fitted returns are generated from Fama and MacBeth (1973) estimates.



with the estimates for the original test assets.

6 Conclusions

A large recent literature has attemped to explain the size and value effects with extenstions of the standard CAPM or consumption CAPM. These model can be categorized as either: (1) conditional versions of the CAPM; (2) conditional versions of the consumption CAPM, or (3) alternative factor models. These models are then tested with size and book-to-market sorted portfolios. These tests tend to support the proposed models, in the sense that the tests fail to reject the null hypothesis that the return data are consistent with the model. These findings, however, are somewhat disconcerting since, as we show in Section

3, the proposed factors have very low pairwise correlations and generate very different estimates of expected returns of individual assets.

As we argue, the tests do not fail to reject the models because the models are all correct – given the low correlations of the proposed factors they cannot all be correct – but rather because the tests have very little power to reject. More specifically, using logic similar to the arguments in Daniel and Titman (1997), we argue that for the tests to have power, test assets are required that have loadings on the proposed factors that are not highly correlated with the test asset characteristics (size and book-to-market ratios).

To illustrate this point we examine two specific models; the model developed in Campbell and Vuolteenaho (2004) and the model Lettau and Ludvigson (2001). As we show the empirical tests presented in these papers look very different when the test portfolios are formed either on the basis of predicted factor loadings or industry affiliation.

We tend to agree with Lewellen, Nagel, and Shanken (2010), who argue that these models should ultimately be tested on individual stock returns rather than portfolios. Of course, existing tests examine portfolios rather than individual stocks because of the numerous challenges associated with examining a sample of several thousand stocks over a sample period of several hundred months. In our future research we hope to address this challenge.

Appendix A. Conditional Models and Conditional Tests

This appendix reviews results on conditional and unconditional models and tests of models.

A.1. Conditional and Unconditional Factor Models

In the absence of arbitrage, all assets are priced by a pricing kernel $\tilde{m}$ such that:

$$E_t[\tilde{m}_{t+1}\tilde{R}_{t+1}] = 1.$$

An *unconditional* k-factor model specifies that the pricing kernel is a linear function of a set of factors:

$$\tilde{m}_{t+1} = a + \mathbf{b}\,\tilde{\mathbf{f}}_{t+1} \tag{5}$$

where a and $\mathbf{b}$ are time-invariant. In contrast, a *conditional* k-factor model specifies that:

$$\tilde{m}_{t+1} = a_t + \mathbf{b}_t'\,\tilde{\mathbf{f}}_{t+1}$$

Here, in contrast to the specification in equation (5), a_t and $\mathbf{b}_t$ are not time invariant, but are adapted to the time t information set.

To test a conditional factor model, we generally specify that a_t and $\mathbf{b}_t$ are linear functions of a $(m \times 1)$ vector of instruments $\mathbf{Z}_t \in \mathcal{F}_t$:

$$\begin{aligned} a_t &= \mathbf{a}'\,\mathbf{Z}_t \\ \mathbf{b}_t &= \mathbf{b}\mathbf{Z}_t \end{aligned}$$

where $\mathbf{a}$ is $(m \times 1)$ and $\mathbf{b}$ is $(k \times m)$. This gives:

$$\tilde{m}_{t+1} = \mathbf{a}'\mathbf{Z}_t + (\mathbf{b}\mathbf{Z}_t)'\,\tilde{\mathbf{f}}_{t+1}$$

A.1..1 Interpreting Conditional Factor Models

As noted by Cochrane (2000), a conditional k-factor model with m conditioning variables is equivalent to a unconditional factor model with $(k \cdot m)$ factors.

For example, the unconditional CAPM specifies that:

$$\tilde{m}_{t+1} = a + b\tilde{r}_{m,t+1},$$

where a and b are time invariant. The Lettau and Ludvigson (2001) conditional CAPM specifies that

$$\tilde{m}_{t+1} = (\gamma_0 + \gamma_1 z_t) + (\eta_0 + \eta_1 z_t)\tilde{r}_{m,t+1} \tag{6}$$

where, in their quarterly tests, the instrument z_t is their *cay* variable measured at the start of the quarter. Notice that this model has the implication that, for a $(N \times 1)$ vector of asset returns from t to $t+1$, and given an observable risk-free rate:

$$(\tilde{\mathbf{r}}_{t+1} - \mathbf{1} r_{f,t+1}) = \boldsymbol{\beta}_m(\tilde{r}_{m,t+1} - r_{f,t+1}) + \boldsymbol{\beta}_{mz}(\tilde{r}_{m,t+1} - r_{f,t+1}) z_t + \tilde{\boldsymbol{\epsilon}}_t \tag{7}$$

where $\tilde{\mathbf{r}}$, $\mathbf{1}$, $\boldsymbol{\beta}_m$, $\boldsymbol{\beta}_{mz}$, and $\tilde{\boldsymbol{\epsilon}}_t$ and $(N \times 1)$ vectors, and $r_{f,t+1}$ is the return on an efficient portfolio uncorrelated with the market portfolio return – it is the risk-free rate if it exists, or the (stochastic) return on a minimum-variance zero-beta portfolio.

Either equation (6) or equation (7) shows that this conditional CAPM is equivalent to a two factors model with factors equal to:

1. The excess market return, defined as the profit that results from investing \$1 in the market portfolio and shorting \$1 of the risk-free (or zero-beta) asset.
2. The *scaled excess-market return*, defined as the profit that results from investing \$$z_t$ in the market portfolio and shorting \$$z_t$ of the risk-free (or zero-beta) asset.

A.2. Conditional Tests of Factor Models

Any test of a factor model will be a test of the set of moment restrictions:

$$E_t[\tilde{m}_{t+1}\tilde{\mathbf{R}}_{t+1}] = \mathbf{1}. \tag{8}$$

An *unconditional test* examines the moment restriction that results from taking an unconditional expectation of equation (5):

$$E[\tilde{m}_{t+1}\tilde{\mathbf{R}}_{t+1}] = \mathbf{1}.$$

A *conditional test* examines additional restrictions implied by equation (8), specifically, that for any set of instruments $\mathbf{Z}_t$ in $\mathcal{F}_t$:

$$E\left[\left(\tilde{m}_{t+1}\tilde{\mathbf{R}}_{t+1} - \mathbf{1}\right) \otimes \mathbf{Z}_t\right] = \mathbf{0} \tag{9}$$

The set of papers that we consider here perform *unconditional* tests of *conditional* factor models. These papers generally do not test the additional moment restrictions implied by (9). In the language of Cochrane (2000), they don't augment the return space with scaled test assets – but they do augment the set of factors with scaled factors.

References

Aït-Sahalia, Yacine, Jonathan A. Parker, and Motohiro Yogo, 2004, Luxury goods and the equity premium, *Journal of Finance* 59, 2959–3004.

Ang, Andrew, and Joseph Chen, 2007, CAPM over the long-run: 1926-2001, *Journal of Empirical Finance* 14, 1–40.

Breeden, Douglas T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–96.

Campbell, John Y, 1993, Intertemporal asset pricing without consumption data, *American Economic Review* 83, 487–512.

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Campbell, John Y., and Tuomo Vuolteenaho, 2004, Bad beta, good beta, *American Economic Review* 94, 1249–1275.

Cochrane, John H., 1996, A cross-sectional test of an investment-based asset pricing model, *Journal of Political Economy* 104, 572–621.

——— , 2000, *Asset Pricing* (Princeton University Press: NJ).

Cohen, Randolph B., Christopher Polk, and Tuomo Vuolteenaho, 2003, The value spread, *The Journal of Finance* 58, 609–642.

Daniel, Kent D., and Sheridan Titman, 1997, Evidence on the characteristics of cross-sectional variation in common stock returns, *Journal of Finance* 52, 1–33.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

——— , 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

——— , 1995, Size and book-to-market factors in earnings and returns, *Journal of Finance* 50, 131–156.

——— , 1997, Industry costs of equity, *Journal of Financial Economics* 43, 153–193.

Fama, Eugene F., and James MacBeth, 1973, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Ferson, Wayne E., and Campbell R. Harvey, 1999, Conditioning variables and the cross section of stock returns, *Journal of Finance* 54, 1325 – 1360.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121–1152.

Hansen, Lars, and Scott F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, *Econometrica* 55, 587–613.

Hansen, Lars P., 1982, Large sample properties of generalized method of moments estimators, *Econometrica* 50, 1029–1054.

———, and Ravi Jagannathan, 1991, Implications of security market data for models of dynamic economies, *Journal of Political Economy* 99, 225–262.

Harvey, Campbell R., 1989, Time-varying conditional covariances in tests of asset pricing models, *Journal of Financial Economics* 24, 289–317.

Heaton, John C., and Deborah J. Lucas, 2000, Portfolio choice and asset prices: The importance of entrepeneurial risk, *Journal of Finance* 55, 1163–1198.

Hodrick, Robert J., and Xiaoyan Zhang, 2001, Evaluating the specification errors of asset pricing models, *Journal of Financial Economics* 62, 327–376.

Jagannathan, Ravi, and Zhenyu Wang, 1996, The CAPM is alive and well, *Journal of Finance* 51, 3–53.

Lettau, Martin, and Sydney Ludvigson, 2001, Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying, *Journal of Political Economy* 109, 1238–1287.

Lewellen, Jonathan, and Stefan Nagel, 2006, The conditional CAPM does not explain asset-pricing anomalies, *Journal of Financial Economics* 58, 289–314.

———, and Jay Shanken, 2010, A skeptical appraisal of asset pricing tests, *Journal of Financial Economics* 96, 175–194.

Li, Qing, Maria Vassalou, and Yuhang Xing, 2002, An investment-growth asset pricing model, Columbia GSB working paper.

Lintner, John, 1965, The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13–37.

Lustig, Hanno N., and Stijn G. Van Nieuwerburgh, 2005, Housing collateral, consumption insurance, and risk premia: An empirical perspective, *The Journal of Finance* 60, 1167–1219.

Merton, Robert C., 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.

Parker, Jonathan A., and Christian Julliard, 2005, Consumption risk and the cross section of expected returns, *Journal of Political Economy* 113, 185–222.

Petkova, Ralitsa, 2006, Do the fama-french factors proxy for innovations in predictive variables?, *Journal of Finance* 61, 581–612.

———, and Lu Zhang, 2005, Is value riskier than growth?, *Journal of Financial Economics* 78, 187–202.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel, 2007, Housing, consumption and asset pricing, *Journal of Financial Economics* 83, 531–569.

Santos, Tano, and Pietro Veronesi, 2005, Labor income and predictable stock returns, *Review of Financial Studies* 19, 1–44.

Sharpe, William F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Table 1: **Proposed Factor Models**

This table lists a subset of the factor models examined in the finance literature, and the factors and conditioning variables considered in these tests.

Paper	Factor(s)	Cond. Vars.
Conditional (C)CAPM Models		
Ferson and Harvey (1999)	VW	S&P 500 Dividend Yield[2]
Lettau and Ludvigson (2001)	VW or Cons Growth	*cay*
Santos and Veronesi (2005)	VW + Labor Income Growth	Labor Income to Cons Ratio (s)
Petkova and Zhang (2005)	VW Index	$E[R_m]$ based on BC Vars
Alternative-Factor Models		
Fama and French (1993)	VW, HML, SMB	
Jagannathan and Wang (1996)	Labor Income Growth	DEF
Heaton and Lucas (2000)	Proprietary Income Growth	
Piazzesi, Schneider, and Tuzel (2007)	Cons Growth $+\Delta$ NH Expenditure Ratio ($\Delta log(\alpha)$)	Non-Housing Expenditure Ratio (α)
Lustig and Van Nieuwerburgh (2005)	Scaled Rental Price Change ($A\Delta log\rho$)	Housing Collateral Ratio
Aït-Sahalia, Parker, and Yogo (2004)	Luxury Good Consumption	
Li, Vassalou, and Xing (2002)	Sector Inv. Growth Rates	
Parker and Julliard (2005)	Innovations in Future Long Horizon Consumption Growth	
Campbell and Vuolteenaho (2004)	CF and DR news	
Petkova (2006)	Innovations in DIV, DEF, TERM, and T-Bill	

Table 2: **Correlations of Candidate Factors**

This table presents the sample correlation matrix of a subset of the factors that have been proposed in the literature as explanations for the value premium. Each of the conditioning variables (DP, $\widehat{cay}$ and s) is demeaned. The sample correlations are each estimated using quarterly data over the period 1963Q4:1998Q3.

	HML	DP$\cdot r_m$	$\widehat{\text{cay}}\cdot r_m$	$s\cdot r_m$	$\widehat{\text{cay}}\cdot\Delta c$	Δy	Δ(prop)	$\Delta\log(\alpha)$	$-N_{CF}$
HML	1	-0.10	0.07	-0.05	0.06	0.01	0.07	0.11	0.27
DP$\cdot r_m$	-0.10	1	0.61	0.37	0.14	-0.01	0.04	0.00	-0.09
$\widehat{\text{cay}}\cdot r_m$	0.07	0.61	1	0.03	0.12	-0.03	-0.16	-0.00	-0.12
$s\cdot r_m$	-0.05	0.37	0.03	1	0.07	0.03	0.14	-0.07	0.07
$\widehat{\text{cay}}\cdot\Delta c$	0.06	0.14	0.12	0.07	1	0.13	0.10	-0.07	0.06
Δy	0.01	-0.01	-0.03	0.03	0.13	1	0.25	0.15	-0.10
Δ(prop)	0.07	0.04	-0.16	0.14	0.10	0.25	1	0.28	0.11
$\Delta\log(\alpha)$	0.11	0.00	-0.00	-0.07	-0.07	0.15	0.28	1	0.09
$-N_{CF}$	0.27	-0.09	-0.12	0.07	0.06	-0.10	0.11	0.09	1

Table 3: **Simulation Test Results Summary**

The first two rows of this table present the results of OLS regressions of $E[r_i] = \lambda_0 + \lambda_1\beta_i + u_i$ for the simulated data, sorted into 10 portfolios according to BM (first row), and factor β (second row). The third row of the table shows the results of the OLS regression test for the 100 portfolios sorted first into deciles based on BM, and then into sub-portfolios based on on factor β. Here the regression is:

$$E[r_i] = \lambda_0 + \lambda_1\beta_i + \gamma_2 I(2) + \cdots + \gamma_{10} I(10) + u_i$$

where $I(2)$ is an indicator variables which is 1 if the firm is in BM decile 2, and zero otherwise.

	$\hat{\lambda}_0$	$\hat{\lambda}_1$	$R^2(\%)$
BM Sort	0.1006 (11.6)	0.0946 (39.8)	94.4%
β Sort	0.1010 (187.0)	0.0097 (17.4)	97.4%
Multiple Sort*	0.0463 (92.5)	0.0005 (3.1)	99.8%

*The coefficients and t-statistics associated with the 9 indicator variables are not shown for this regression.

Table 4: **BM/Size/Pre-β_{Mkt} Portfolios - $\bar{r}$s and Post-Formation βs**

For the 1933:07-1963:06 period, this table presents the average monthly returns (in %/month) and the post-formation market βs and the corresponding t-statistics for portfolios formed on the basis of independent sorts into 3 portfolios each based on size and book-to-market ratio, followed by dependent sorts into 5 sub-portfolios based on pre-formation betas. Size and book-to-market sorts are based on NYSE cutoffs. The final column of the table, labeled $5-1$, gives the average return and β for the zero-investment portfolio formed by buying $1 of the high β portfolio, and selling $1 of the low β sub-portfolio. The final row of the table, labeled "avg port.," gives the statistics for the equal-weighted portfolio of the 9 sub-portfolios listed directly above.

Chr Pt		$\bar{r}$ (%/mo)					$t(\bar{r})$					$\bar{r}$	$t(\bar{r})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	1.22	1.18	1.19	0.85	0.81	(3.48)	(2.54)	(2.57)	(1.47)	(1.39)	-0.40	(-0.96)
1	2	1.40	1.28	1.41	1.17	1.24	(4.40)	(3.42)	(3.38)	(2.32)	(2.41)	-0.16	(-0.51)
1	3	1.33	1.47	1.68	1.56	1.44	(3.91)	(3.36)	(3.26)	(3.19)	(2.36)	0.11	(0.30)
2	1	1.02	0.98	1.07	0.98	0.94	(4.40)	(3.45)	(3.03)	(2.56)	(2.18)	-0.08	(-0.28)
2	2	1.23	1.20	1.28	1.37	1.18	(5.09)	(4.03)	(3.71)	(3.56)	(2.57)	-0.05	(-0.18)
2	3	1.09	1.30	1.32	1.34	1.28	(3.53)	(3.21)	(3.11)	(2.75)	(2.39)	0.19	(0.54)
3	1	0.70	0.86	0.86	1.07	0.96	(3.46)	(3.75)	(3.16)	(3.50)	(2.77)	0.26	(1.19)
3	2	0.93	1.07	1.27	1.06	0.99	(4.37)	(4.02)	(3.99)	(3.14)	(2.47)	0.06	(0.20)
3	3	0.99	1.11	1.24	1.31	1.26	(3.14)	(2.94)	(2.87)	(2.92)	(2.48)	0.27	(0.88)
avg prt		1.10	1.16	1.26	1.19	1.12	(4.42)	(3.68)	(3.48)	(2.93)	(2.48)	**0.02**	**(0.09)**

Chr Prt		$\hat{\beta}_{Mkt}$					$t(\hat{\beta}_{Mkt})$					$\hat{\beta}_{Mkt}$	$t(\hat{\beta}_M)$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	1.03	1.40	1.30	1.71	1.76	(22.09)	(23.13)	(19.99)	(22.09)	(22.96)	0.73	(9.50)
1	2	1.06	1.24	1.40	1.66	1.70	(31.88)	(30.99)	(33.38)	(30.01)	(30.81)	0.64	(11.94)
1	3	1.07	1.40	1.66	1.61	1.88	(25.87)	(27.62)	(28.25)	(30.35)	(25.03)	0.82	(14.15)
2	1	0.78	0.99	1.25	1.36	1.53	(32.85)	(39.48)	(42.87)	(42.84)	(41.48)	0.75	(17.82)
2	2	0.82	1.05	1.24	1.40	1.63	(34.11)	(42.33)	(47.79)	(50.05)	(43.75)	0.81	(20.53)
2	3	0.99	1.39	1.47	1.63	1.80	(27.17)	(36.04)	(36.88)	(32.82)	(32.45)	0.81	(14.11)
3	1	0.70	0.82	0.99	1.13	1.29	(36.81)	(48.07)	(53.82)	(58.54)	(59.26)	0.58	(18.06)
3	2	0.70	0.89	1.11	1.21	1.40	(30.50)	(32.76)	(40.13)	(45.52)	(40.22)	0.70	(14.70)
3	3	1.04	1.27	1.44	1.56	1.71	(31.20)	(32.06)	(31.45)	(37.86)	(33.24)	0.66	(12.75)
avg prt		0.91	1.16	1.32	1.47	1.63	(53.20)	(56.75)	(53.61)	(50.75)	(47.68)	**0.72**	**(23.80)**

Table 5: **Early Period Times Series Regression Intercepts**

This table presents the results of the time-series regressions of the realized excess returns and t-statistics of the 45 portfolios on the realized excess returns of the CRSP value-weighted portfolio returns over the 1933:07-1963:06 period. The left part of the table reports the estimated regression intercepts, and the right part presents the t-statistics associated with these intercepts. The last row of the table gives the intercepts and t-statistics for the average portfolio, and the last two columns give the estimated intercepts and t-stats for the $5-1$ difference portfolio, as described in the text.

Chr Prt		$\hat{\alpha}$					$t(\hat{\alpha})$					$\hat{\alpha}$	$t(\hat{\alpha})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.24	-0.15	*-0.05*	-0.78	-0.86	(1.01)	(-0.50)	*(-0.17)*	(-2.02)	(-2.26)	-1.10	(-2.88)
1	2	0.39	0.10	*0.07*	-0.41	-0.38	(2.34)	(0.50)	*(0.33)*	(-1.49)	(-1.39)	-0.77	(-2.87)
1	3	0.32	0.13	*0.10*	0.02	-0.36	(1.54)	(0.53)	*(0.33)*	(0.09)	(-0.96)	-0.67	(-2.34)
2	1	0.28	0.03	*-0.13*	-0.31	-0.51	(2.34)	(0.27)	*(-0.87)*	(-2.00)	(-2.80)	-0.79	(-3.79)
2	2	0.45	0.19	*0.09*	0.04	-0.38	(3.75)	(1.57)	*(0.73)*	(0.27)	(-2.04)	-0.83	(-4.20)
2	3	0.15	-0.03	*-0.08*	-0.22	-0.43	(0.85)	(-0.14)	*(-0.39)*	(-0.89)	(-1.57)	-0.59	(-2.05)
3	1	0.03	0.07	*-0.09*	-0.01	-0.26	(0.36)	(0.81)	*(-0.96)*	(-0.08)	(-2.43)	-0.30	(-1.84)
3	2	0.26	0.22	*0.20*	-0.09	-0.35	(2.27)	(1.61)	*(1.48)*	(-0.71)	(-2.03)	-0.61	(-2.57)
3	3	-0.01	-0.09	*-0.13*	-0.18	-0.37	(-0.05)	(-0.48)	*(-0.58)*	(-0.86)	(-1.46)	-0.36	(-1.40)
avg prt		0.23	0.05	*-0.00*	-0.22	-0.43	(2.75)	(0.52)	*(-0.01)*	(-1.49)	(-2.55)	**-0.67**	**(-4.42)**

Table 6: **Early Period Post-Formation CF and DR betas**

This table presents the estimated CF and DR betas from time-series regressions of the realized excess returns and t-statistics of the 45 portfolios on the component of the market return attributable to cash-flow and discount-rate news, as calculated by Campbell and Vuolteenaho (2004), over the 1933:07-1963:06 period. The left parts of the two panels report CF and DR betas, and the right parts of each panel present the t-statistics associated with these betas. The last row of the table gives the betas and t-statistics for the average portfolio, and the last two columns give the estimated betas and t-stats for the 5−1 difference portfolio, as described in the text.

Chr	Prt	$\hat{\beta}_{CF}$					$t(\hat{\beta}_{CF})$					$\hat{\beta}_{CF}$	$t(\hat{\beta}_{CF})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.20	0.26	0.26	0.38	0.45	(4.98)	(4.97)	(4.91)	(5.95)	(7.03)	0.26	(5.44)
1	2	0.22	0.28	0.29	0.36	0.42	(6.15)	(6.82)	(6.34)	(6.42)	(7.57)	0.21	(5.94)
1	3	0.31	0.35	0.42	0.41	0.47	(8.49)	(7.39)	(7.54)	(7.75)	(7.07)	0.16	(4.05)
2	1	0.13	0.18	0.24	0.26	0.27	(4.94)	(5.56)	(6.01)	(6.15)	(5.49)	0.14	(4.27)
2	2	0.18	0.24	0.25	0.30	0.34	(6.81)	(7.47)	(6.70)	(7.11)	(6.67)	0.16	(4.84)
2	3	0.28	0.37	0.39	0.43	0.45	(8.45)	(8.53)	(8.68)	(8.24)	(7.75)	0.17	(4.27)
3	1	0.09	0.13	0.16	0.16	0.24	(3.73)	(4.97)	(5.34)	(4.75)	(6.16)	0.15	(6.24)
3	2	0.13	0.21	0.21	0.25	0.29	(5.34)	(7.26)	(5.99)	(6.70)	(6.45)	0.16	(4.70)
3	3	0.22	0.31	0.32	0.34	0.42	(6.28)	(7.67)	(6.75)	(6.96)	(7.66)	0.20	(5.88)
avg prt		0.19	0.26	0.28	0.32	0.37	(7.13)	(7.57)	(7.19)	(7.29)	(7.56)	**0.18**	**(6.77)**

Chr	Prt	$\hat{\beta}_{DR}$					$t(\hat{\beta}_{DR})$					$\hat{\beta}_{DR}$	$t(\hat{\beta}_{DR})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.84	0.99	1.02	1.13	1.27	(11.91)	(10.07)	(10.57)	(9.04)	(10.37)	0.43	(4.46)
1	2	0.77	0.91	1.02	1.11	1.19	(12.08)	(12.09)	(12.31)	(10.57)	(11.36)	0.42	(6.02)
1	3	0.82	1	1.08	1.15	1.24	(11.91)	(11.06)	(9.91)	(11.48)	(9.55)	0.42	(5.18)
2	1	0.63	0.72	0.91	0.96	1.02	(14.19)	(12.86)	(13.15)	(12.65)	(11.55)	0.39	(6.09)
2	2	0.62	0.75	0.89	0.97	1.13	(13.04)	(12.77)	(13.14)	(12.69)	(12.35)	0.51	(8.22)
2	3	0.68	0.86	0.93	1.04	1.17	(10.60)	(10.06)	(10.51)	(10.17)	(10.43)	0.48	(6.11)
3	1	0.54	0.61	0.69	0.82	0.90	(13.87)	(13.81)	(13.05)	(13.93)	(13.21)	0.36	(7.35)
3	2	0.49	0.52	0.75	0.79	0.91	(11.38)	(9.04)	(11.56)	(11.40)	(11.05)	0.42	(6.21)
3	3	0.65	0.86	0.91	1.01	1.08	(9.73)	(11.08)	(9.96)	(10.90)	(10.15)	0.43	(6.20)
avg prt		0.67	0.80	0.91	1.00	1.10	(14.12)	(12.87)	(12.78)	(12.29)	(12.11)	**0.43**	**(8.25)**

Table 7: **Late Period Post-Formation CF and DR betas**

This table presents the estimated CF and DR betas from time-series regressions of the realized excess returns and t-statistics of the 45 portfolios on the component of the market return attributable to cash-flow and discount-rate news, as calculated by Campbell and Vuolteenaho (2004), over the 1963:07-2001:12 period. The left parts of the two panels report CF and DR betas, and the right parts of each panel present the t-statistics associated with these betas. The last row of the table gives the betas and t-statistics for the average portfolio, and the last two columns give the estimated betas and t-stats for the 5−1 difference portfolio, as described in the text.

Chr Prt		$\hat{\beta}_{CF}$					$t(\hat{\beta}_{CF})$					$\hat{\beta}_{CF}$	$t(\hat{\beta}_{CF})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.090	0.110	0.119	0.136	0.157	(2.62)	(3.04)	(3.04)	(3.08)	(3.25)	0.067	(2.54)
1	2	0.117	0.127	0.134	0.153	0.181	(4.55)	(4.46)	(4.16)	(4.20)	(4.22)	0.064	(2.63)
1	3	0.133	0.153	0.170	0.160	0.211	(4.89)	(5.25)	(5.24)	(4.47)	(5.12)	0.078	(3.63)
2	1	0.077	0.074	0.091	0.099	0.130	(2.82)	(2.39)	(2.72)	(2.74)	(2.86)	0.053	(1.93)
2	2	0.106	0.110	0.135	0.158	0.157	(4.71)	(4.14)	(4.94)	(5.19)	(4.23)	0.051	(2.10)
2	3	0.115	0.134	0.141	0.173	0.214	(4.83)	(4.99)	(4.71)	(5.41)	(5.49)	0.098	(3.54)
3	1	0.061	0.058	0.068	0.074	0.091	(2.49)	(2.27)	(2.42)	(2.41)	(2.50)	0.031	(1.27)
3	2	0.052	0.075	0.100	0.118	0.146	(2.28)	(3.04)	(3.74)	(4.15)	(4.39)	0.094	(3.70)
3	3	0.084	0.099	0.127	0.123	0.161	(3.50)	(4.00)	(4.48)	(3.95)	(4.67)	0.077	(2.90)
avg prt		0.093	0.104	0.121	0.133	0.161	(4.15)	(4.18)	(4.32)	(4.32)	(4.40)	0.068	(3.54)

Chr Prt		$\hat{\beta}_{DR}$					$t(\hat{\beta}_{DR})$					$\hat{\beta}_{DR}$	$t(\hat{\beta}_{DR})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	1.188	1.267	1.386	1.542	1.658	(17.05)	(17.46)	(17.77)	(17.32)	(16.82)	0.470	(7.28)
1	2	0.859	0.962	1.117	1.237	1.466	(15.90)	(16.07)	(16.92)	(16.37)	(16.55)	0.607	(10.79)
1	3	0.852	0.929	1.037	1.188	1.356	(14.22)	(14.56)	(14.61)	(15.71)	(15.32)	0.504	(9.93)
2	1	0.927	1.108	1.221	1.324	1.642	(16.71)	(18.46)	(18.78)	(18.85)	(18.47)	0.715	(11.33)
2	2	0.669	0.903	0.948	1.030	1.302	(13.35)	(16.44)	(16.63)	(15.91)	(17.34)	0.632	(11.49)
2	3	0.608	0.786	0.952	1.006	1.217	(10.80)	(12.89)	(14.65)	(14.28)	(14.11)	0.609	(9.11)
3	1	0.780	0.877	1.020	1.132	1.304	(15.21)	(17.21)	(18.72)	(19.45)	(18.25)	0.524	(9.16)
3	2	0.551	0.697	0.870	0.878	1.128	(10.41)	(12.69)	(15.27)	(14.07)	(16.35)	0.577	(9.52)
3	3	0.500	0.583	0.759	0.947	1.059	(8.58)	(9.97)	(11.56)	(13.81)	(13.93)	0.560	(8.71)
avg prt		0.770	0.901	1.034	1.143	1.348	(16.80)	(18.15)	(18.90)	(19.05)	(18.79)	0.577	(13.67)

Table 8: **Late Period Average Portfolio Returns**

For the 1963:07-2001:12 period, this table presents the average monthly returns (in %/month) and the corresponding t-statistics for portfolios formed on the basis of independent sorts into 3 portfolios each based on size and book-to-market ratio, followed by dependent sorts into 5 sub-portfolios based on pre-formation cash-flow betas. Size and book-to-market sorts are based on NYSE cutoffs. The final column of the table, labeled $5-1$, gives the average return for the zero-investment portfolio formed by buying \$1 of the high β portfolio, and selling \$1 of the low β sub-portfolio. The final row of the table, labeled "avg port.," gives the average return and t-statistic for the equal-weighted portfolio of the 9 sub-portfolios listed directly above.

Chr Prt		$\bar{r}$ (%/mo)					$t(\bar{r})$					$\bar{r}$	$t(\bar{r})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.53	0.57	0.77	0.53	0.24	(1.81)	(1.87)	(2.30)	(1.40)	(0.59)	-0.29	(-1.28)
1	2	0.71	0.85	0.88	0.76	0.95	(3.23)	(3.45)	(3.18)	(2.44)	(2.57)	0.23	(1.12)
1	3	1.04	0.95	1.07	1.02	1.11	(4.40)	(3.75)	(3.78)	(3.31)	(3.10)	0.07	(0.39)
2	1	0.66	0.64	0.55	0.42	0.49	(2.87)	(2.45)	(1.92)	(1.37)	(1.27)	-0.17	(-0.74)
2	2	0.63	0.62	0.80	0.66	0.93	(3.26)	(2.71)	(3.39)	(2.49)	(2.92)	0.29	(1.44)
2	3	0.67	0.90	1.02	0.95	1.02	(3.25)	(3.88)	(3.97)	(3.43)	(3.02)	0.35	(1.48)
3	1	0.57	0.55	0.54	0.48	0.52	(2.75)	(2.54)	(2.27)	(1.87)	(1.67)	-0.05	(-0.25)
3	2	0.51	0.61	0.48	0.57	0.77	(2.67)	(2.94)	(2.10)	(2.32)	(2.69)	0.25	(1.17)
3	3	0.67	0.57	0.73	0.75	0.79	(3.29)	(2.70)	(3.00)	(2.80)	(2.65)	0.11	(0.50)
avg prt		0.67	0.70	0.76	0.68	0.76	(6.26)	(5.73)	(5.38)	(4.60)	(4.10)	0.09	(0.54)

Table 9: **Tests of the Lettau and Ludvigson (2001) Consumption CAPM Using Size/BM sorted and Industry Portfolios**
This table presents estimates of the coefficients, t-statistics (in parentheses) and R^2s from a second stage Fama and MacBeth (1973) regression. The data is monthly, and covers the sample period 1963:10 - 1998:09. The factors $\widehat{cay}_t$, Δc_{t+1}, and $\widehat{cay}_t \cdot \Delta c_{t+1}$ are those used in the corresponding regressions in Lettau and Ludvigson (2001). The test assets are (1) the 25 size-book-to-market sorted portfolios of Fama and French (1993), and (2) the industry portfolios from Fama and French (1997). All of the portfolio return data was obtained from Kenneth French's web site.

Ports	Const	$\widehat{cay}_t$	Δc_{t+1}	$\widehat{cay}_t \cdot \Delta c_{t+1}$	R^2
25 FF SZ/BM	4.28	-0.12	0.02	0.0057	0.70
	(11.36)	(-0.66)	(0.23)	(3.10)	
48 FF Indust.	2.94	0.27	-0.10	0.0002	0.30
	(14.49)	(2.99)	(-2.26)	(0.24)	
38 FF Indust.	3.13	0.18	-0.07	0.0003	0.09
	(8.40)	(0.93)	(-0.92)	(0.17)	
11 FF Indust.	2.91	-0.02	0.03	-0.0033	0.51
	(7.08)	(-0.09)	(0.29)	(-1.84)	